ZERO SEQUENCE IMPEDANCE OF OVERHEAD TRANSMISSION LINES

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Abstract - **This paper reviews the basic equations for ground loop current flow, including showing how neutral wires are included in the equations, then showing how this impedance is transformed from an ABC domain impedance to the 012 domain impedance. A side benefit of the approach taken is that the paper shows how one calculates the sequence impedances of untransposed power lines, including calculation of the off-diagonal (mutual) elements of the sequence component 012 domain impedances. The paper also addresses the calculation of mutual impedances between two parallel lines.**

Keywords - **zero sequence, positive sequence, negative sequence, mutual impedance**

I. INTRODUCTION

The paper begins by analyzing system impedances in the ABC (physical or phase) domain, first without any overhead ground wires, and then shows how the overhead ground conductors are incorporated into the analysis. Then the paper shows the translation of these ABC domain impedances to the 012 (sequence) domain, which provides the zero sequence impedances, and other sequence impedances. Thereafter, the paper discusses the calculation of mutual impedance coupling between two parallel lines.

II. BASIC ABC DOMAIN IMPEDANCE CONCEPTS

In Fig. 1 there are three phase current loops, A, B, and C, each passing through a common neutral/ground. For this initial investigation, there is no independent metallic neutral/ground conductor, so the phase currents sum together and return through the earth in the current named I_G.

The phase A loop is shown as a dotted line. Each phase loop in Fig. 1 has a different impedance; each loop is defined by a different current path, a different loop cross section area and, when magnetic core material is involved, a different permeability of the material through which the flux passes. To keep the drawing from becoming exceedingly complex, only the main representative flux loops are shown and only for phase A. One's imagination should be used to fill in the blanks. An inspection of Fig. 1 shows that we have 3 current loops, but 4 currents are shown on the diagram. This means we can actually only set up 3 voltage drop equations, and we will need to take advantage of $I_G = I_A + I_B + I_C$ to remove I_G from directly being part of the voltage drop equations.

Note the assumed direction for positive I_G in the drawing; note that it is opposite on the page (pointing left) relative to current in the wires (pointing right). In some text and articles, the reference direction for I_G is the same direction as in the wires, which, in turn, causes negative signs to appear in some of the voltage drops equations or impedances, which do not appear herein.

To understand the impedances that represent the system in Fig. 1, one first should understand the sources of flux that can be found in loop A in Fig. 1. The flux in loop A is the summation of the flux from several sources:

$$\phi_A = \phi_{AA,S} + \phi_{AA,M} + \phi_{AB} + \phi_{AC} + \phi_{AG} \tag{1}$$

Defining terms in (1):

$\phi_{AA,S}$ = Part of "self" flux. This is flux in loop A due to I_A that is only seen by loop A. Large self flux can be generated by magnetic core devices, e.g., the magnetic core device shown in the diagram has flux that is seen only by loop A and not by loop B or C.

$\phi_{AA,M}$ = Another part of self flux, but is flux in loop A due to I_A but that is also seen by loop B and/or C. This flux is the source of mutual inductance of phase A to phases B and C.

ϕ_{AB} = Flux in loop A due to current I_B. This is the source of mutual coupling between I_B and phase A.

ϕ_{AC} = Flux in loop A due to current I_C. This is the source of mutual coupling between I_C and phase A.

ϕ_{AG} = Flux in loop A due to current I_G. This is the source of mutual coupling between I_G and phase A.

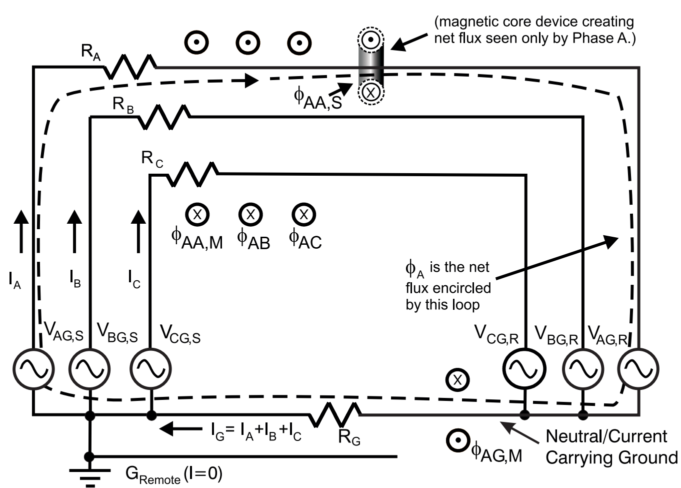

Fig. 1. Phase A Voltage Loop and Flux when part of a Three Phase System

In an even more complete drawing, not shown in Fig. 1, the current in I_G would be modeled via 3 different underground conductors. The current in the ground flows in a slightly different path depending on whether the source for I_G is I_A, I_B, or I_C, which will result in a system of 6 currents rather than 4, and which increases the number of mutual flux components and greatly increases the complexity of analysis. We will proceed on the assumption of a single common ground return path as "close enough" for our investigation.

The flux distribution indicated by Fig. 1 can be restated in terms of inductance, L, and ground current can be removed from the equations by applying $i_g = i_a + i_b + i_c$:

$$\begin{aligned}\phi_A &= L_{AA} i_A(t) + L_{AB} i_B(t) + L_{AC} i_C(t) + L_{AG} i_G(t) \\ &= L_{AA} i_A(t) + L_{AB} i_B(t) + L_{AC} i_C(t) + L_{AG}\left(i_A(t) + i_B(t) + i_C(t)\right) \\ &= \left(L_{AA} + L_{AG}\right) i_A(t) + \left(L_{AB} + L_{AG}\right) i_B(t) + \left(L_{AC} + L_{AG}\right) i_C(t)\end{aligned} \quad . \qquad (2)$$

This flux distribution, when coupled with resistive voltage drop, becomes the basis for system impedances. There are two basic parts of a line's impedance: "self" impedance and "mutual" impedance (we are referring to mutual impedances between phase currents; we are not yet referring to mutual impedance between adjacent lines). Self impedance refers to the voltage drop in a phase loop due to current in that same phase. For instance, Z_{AA} relates the voltage loss in loop A due to I_A. The instantaneous voltage equation for current only in phase A is

$$v_{AG,S}(t) - v_{AG,R}(t) = \left(R_A + R_G\right) i_A(t) + \left(L_{AA} + L_{AG}\right)\frac{d}{dt} i_A(t) . \qquad (3)$$

In power system analysis, we are primarily interested in the fundament frequency (50 or 60Hz typically) component, which allows us to restate (3) in terms of $X = j\omega L$ and drop the *d/dt* term. The resultant equation is referred to as phasor analysis and allows one to analyze the system voltage drops using complex number math. Restating (3) in phasor analysis terms gives us

$$V_{AG,S} - V_{AG,R} = \left(R_A + R_G\right) I_A + j\left(X_{AA} + X_{AG}\right) I_A \ . \qquad (4)$$

The equation for self impedance becomes

$$\begin{aligned}Z_{AA} &= \frac{V_{AG,S} - V_{AG,R}}{I_A} \\ &= \left(R_A + R_G\right) + j\left(X_{AA} + X_{AG}\right)\end{aligned} \quad . \qquad (5)$$

Mutual impedance refers to the voltage drop in a loop due to current in another phase. This impedance is mostly inductive, but it also has a resistive component due to a shared neutral wire. For instance, Z_{AB} relates the voltage loss in loop A due to I_B which, in phasor analysis terms, becomes restated to

$$V_{AG,S} - V_{AG,R} = R_G I_B + j\left(X_{AB} + X_{AG}\right) I_B \ . \qquad (6)$$

The equation for mutual impedance becomes

$$\begin{aligned}Z_{AB} &= \frac{V_{AG,S} - V_{AG,R}}{I_B} \\ &= \left(R_G\right) + j\left(X_{AB} + X_{AG}\right)\end{aligned} \quad . \qquad (7)$$

As previously stated, the ground current loop takes slightly different paths depending on the location of the overhead conductor, so phase A, B, and C wires each have a slightly different ground path, but we are assuming that modeling the currents as flowing a single ground conductor is "close enough." To be complete, we would need to have 6 currents, but we will make some allowance for multiple ground paths, as seen below, by allowing for a different R_{AG}+X_{AG}, R_{BG}+X_{BG}. and R_{CG}+X_{CG}. We now have come to the following equation of voltage drop for Fig. 1. (In (8) and all following equations, the "j" term is left as implied in order to simplify the presentation and size of the equation.)

$$\begin{bmatrix} V_{AG,S} \\ V_{BG,S} \\ V_{CG,S} \\ 0 \end{bmatrix} - \begin{bmatrix} V_{AG,R} \\ V_{BG,R} \\ V_{CG,R} \\ 0 \end{bmatrix} = \begin{bmatrix} R_A + X_{AA} & X_{AB} & X_{AC} & R_{AG} + X_{AG} \\ X_{BA} & R_B + X_{BB} & X_{BC} & R_{BG} + X_{BG} \\ X_{CA} & X_{CB} & R_C + X_{CC} & R_{CG} + X_{CG} \\ * & * & * & * \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_G \end{bmatrix} \qquad (8)$$

The *'s in (8) indicate that these elements could not be well defined in this matrix for the system shown in Fig. 1 because the path and impedances that are implied by I_G are already part of the other A, B, and C voltage drop loop equations. In order to eliminate I_G from the equations, note that $I_G = I_A + I_B + I_C$, which means we can rewrite (8) as:

$$\begin{bmatrix} V_{AG,S} \\ V_{BG,S} \\ V_{CG,S} \\ 0 \end{bmatrix} - \begin{bmatrix} V_{AG,R} \\ V_{BG,R} \\ V_{CG,R} \\ 0 \end{bmatrix} = \begin{bmatrix} R_A + X_{AA} & X_{AB} & X_{AC} & R_{AG} + X_{AG} \\ X_{BA} & R_B + X_{BB} & X_{BC} & R_{BG} + X_{BG} \\ X_{CA} & X_{CB} & R_C + X_{CC} & R_{CG} + X_{CG} \\ * & * & * & * \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_A + I_B + I_C \end{bmatrix} \qquad (9)$$

which reduces to

$$\begin{bmatrix} V_{AG,S} \\ V_{BG,S} \\ V_{CG,S} \end{bmatrix} - \begin{bmatrix} V_{AG,R} \\ V_{BG,R} \\ V_{CG,R} \end{bmatrix} = \left[Z_{ABC}\right] \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \qquad (10)$$

where

$$\left[Z_{ABC}\right] = \begin{bmatrix} R_A + R_{AG} + X_{AA} + X_{AG} & R_{AG} + X_{AB} + X_{AG} & R_{AG} + X_{AC} + X_{AG} \\ R_{BG} + X_{BA} + X_{BG} & R_B + R_{BG} + X_{BB} + X_{BG} & R_{BG} + X_{BC} + X_{BG} \\ R_{CG} + X_{CA} + X_{CG} & R_{CG} + X_{CB} + X_{CG} & R_C + R_{BG} + X_{CC} + X_{CG} \end{bmatrix} \qquad (11)$$

An abbreviated way of stating (10) and (11) is

$$\begin{bmatrix} V_{A,S} \\ V_{B,S} \\ V_{C,S} \end{bmatrix} - \begin{bmatrix} V_{A,R} \\ V_{B,R} \\ V_{C,R} \end{bmatrix} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{12}$$

which can be further abbreviated to

$$\mathbf{V_{ABC,S}} - \mathbf{V_{ABC,R}} = \mathbf{Z_{ABC}} \cdot \mathbf{I_{ABC}} \quad . \tag{13}$$

Note in the above equations:

- Wye voltages that reference neutral are implied.
- Positive current is implied to be from the sending end to the receiving end.
- The voltage to remote ground is neither utilized nor calculated explicitly.
- In (12) the "G" reference for voltages is assumed and not explicitly stated. The G reference will be dropped in the rest of the paper.
- Note the implied definition of $\mathbf{Z_{ABC}}$:

$$\mathbf{Z_{ABC}} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix} \tag{14}$$

where:

$Z_{AA} = (V_{AS}\text{-}V_{AR})/ I_A$
$Z_{BA} = (V_{BS}\text{-}V_{BR})/ I_A$
$Z_{CA} = (V_{CS}\text{-}V_{CR})/ I_A$
$Z_{AB} = (V_{AS}\text{-}V_{AR})/ I_B$
$Z_{BB} = (V_{BS}\text{-}V_{BR})/ I_B$
$Z_{CB} = (V_{CS}\text{-}V_{CR})/ I_B$
$Z_{AC} = (V_{AS}\text{-}V_{AR})/ I_C$
$Z_{BC} = (V_{BS}\text{-}V_{BR})/ I_C$
$Z_{CC} = (V_{CS}\text{-}V_{CR})/ I_C$

Note the first subscript letter in $Z_{\#\#}$ refers to the voltage loop, and the second subscript letter refers to the current; e.g., Z_{CB} relates the voltage in the C phase voltage loop due to phase B current. The diagonal terms Z_{AA}, Z_{BB}, and Z_{CC} are referred to as self impedances (e.g., Z_{AA} relates phase A voltage and phase A current), and all other inter-phase quantities are referred to as mutual impedances (e.g., Z_{AB} relates phase A voltage and phase B current).

Fig. 2 is a lumped parameter approach to viewing the impedances described in (14) as well as elsewhere above. As a side note, while a convenient way to show the inductance of an electric circuit is via lumped parameters on the conductors, this method can be technically a bit misleading, possibly giving erroneous perceptions of the nature of inductive impedances. The lumped inductance diagram leaves the impression that a wire by itself defines inductance. Actually, inductance is not fully defined until the conductive route back to the source is defined which, in turn, defines the area of the loop and the flux inside the loop. Some texts provide what appears to be the calculation of the impedance of a wire by itself, but they are actually showing the impedance of the wire for current return some fixed distance away, such as 1 meter or an infinite distance away.

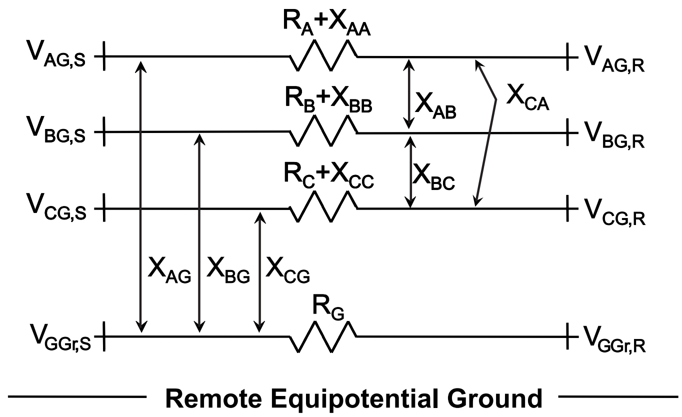

Fig. 2. Lumped Parameter 3 Phase Impedance Network

III. The ABC Impedance Matrix when Independent Ground Conductors are Present

The above equations were based upon a single ground path. In many power lines, if not most, there is one or more ground wires hung along the same right-of-way as the phase conductors. We will use the subscript N to refer to these wires, to avoid confusion with ground current. Also, sometimes a nearby railroad track or similar metallic conductor can also act as a quasi-ground conductor. Each parallel path for the ground current creates an expansion of the impedance matrix. In order to see the effect of a ground wire on the impedance matrix, assume we have two ground paths: a single neutral overhead wire and the ground already discussed in Section II. For this case, to model the ground current path, we need an impedance matrix with a format very similar to (9), but where we add one more line to the matrix for the voltage loop associated with our ground wire. The resultant equation is seen in (15) on the next page.

In (15), note that the applied voltage to the ground conductors is 0, since it is grounded at both ends. Next, note $I_G = I_A + I_B + I_C + I_N$. This matrix can be reduced to a 4x4 matrix in the same approach as was done in (9) and (11), giving (16).

The benefit of applying (16) is that it calculates I_N directly and can be used to determine the current level in the neutral conductor. Further, the concept can be expanded upon to find current in multiple neutral conductors, or even be expanded to calculate the current in each wire of a bundled phase conductor.

The difficulty in applying (16) is that it does not have the format of the standard 3 row $\mathbf{Z_{ABC}}$ impedance matrix we need for classical circuit analysis, where we are primarily interested in A, B, and C currents, and which we will use herein for the calculation of Z_0, Z_1, and Z_2. We need to eliminate I_N from the analysis and reduce (16) to a 3 row equation matrix that calculates only A, B, and C quantities. The method that needs to be used is a matrix manipulation process called Kron Reduction, described in detail in [1], [2], and various linear algebra texts.

$$\begin{bmatrix} V_{A,S} \\ V_{B,S} \\ V_{C,S} \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} V_{A,R} \\ V_{B,R} \\ V_{C,R} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_A + X_{AA} & X_{AB} & X_{AC} & X_{AN} & R_{AG} + X_{AG} \\ X_{BA} & R_B + X_{BB} & X_{BC} & X_{BN} & R_{BG} + X_{BG} \\ X_{CA} & X_{CB} & R_C + X_{CC} & X_{CN} & R_{CG} + X_{CG} \\ X_{NA} & X_{NB} & X_{NC} & R_N + X_{NN} & R_{NG} + X_{NG} \\ * & * & * & * & * \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \\ I_A + I_B + I_C + I_N \end{bmatrix} \tag{15}$$

which becomes, after eliminating the bottom row:

$$\begin{bmatrix} V_{A,S} \\ V_{B,S} \\ V_{C,S} \\ 0 \end{bmatrix} - \begin{bmatrix} V_{A,R} \\ V_{B,R} \\ V_{C,R} \\ 0 \end{bmatrix} = \begin{bmatrix} R_A + R_{AG} + X_{AG} + X_{AA} & R_{AG} + X_{AG} + X_{AB} & R_{AG} + X_{AG} + X_{AC} & R_{AG} + X_{AG} + X_{AN} \\ R_{BG} + X_{BG} + X_{BA} & R_B + R_{BG} + X_{BG} + X_{BB} & R_{BG} + X_{BG} + X_{BC} & R_{BG} + X_{BG} + X_{BN} \\ R_{CG} + X_{CG} + X_{CA} & R_{CG} + X_{CG} + X_{CB} & R_C + R_{CG} + X_{CG} + X_{CC} & R_{CG} + X_{CG} + X_{CN} \\ R_{NG} + X_{NG} + X_{NA} & R_{NG} + X_{NG} + X_{NB} & R_{NG} + X_{NG} + X_{NC} & R_N + R_{NG} + X_{NG} + X_{NN} \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix}. \tag{16}$$

The Kron reduction process might best be seen by an example. Given the 4 row equation

$$\begin{bmatrix} V_{A,S} \\ V_{B,S} \\ V_{C,S} \\ 0 \end{bmatrix} - \begin{bmatrix} V_{A,R} \\ V_{B,R} \\ V_{C,R} \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} & Z_{AN} \\ Z_{BA} & Z_{BB} & Z_{BC} & Z_{BN} \\ Z_{CA} & Z_{CB} & Z_{CC} & Z_{CN} \\ Z_{NA} & Z_{NB} & Z_{NC} & Z_{NN} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_N \end{bmatrix}, \tag{17}$$

to remove the bottom row containing I_N, one applies the analysis process seen here:

$$\begin{bmatrix} V_{A,S} \\ V_{B,S} \\ V_{C,S} \end{bmatrix} - \begin{bmatrix} V_{A,R} \\ V_{B,R} \\ V_{C,R} \end{bmatrix} = \begin{bmatrix} Z_{AA} - \frac{Z_{AN}Z_{NA}}{Z_{NN}} & Z_{AB} - \frac{Z_{AN}Z_{NB}}{Z_{NN}} & Z_{AC} - \frac{Z_{AN}Z_{NC}}{Z_{NN}} \\ Z_{BA} - \frac{Z_{BN}Z_{NA}}{Z_{NN}} & Z_{BB} - \frac{Z_{BN}Z_{NB}}{Z_{NN}} & Z_{BC} - \frac{Z_{BN}Z_{NC}}{Z_{NN}} \\ Z_{CA} - \frac{Z_{CN}Z_{NA}}{Z_{NN}} & Z_{CB} - \frac{Z_{CN}Z_{NB}}{Z_{NN}} & Z_{CC} - \frac{Z_{CN}Z_{NC}}{Z_{NN}} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}. \tag{18}$$

If one had more than one additional ground conductor, this reduction could be repeated the appropriate number of times, gradually reducing a matrix that relates voltage and current on phases A, B, and C plus any number of ground conductors to an equation that simply relates voltage and current on phases A, B, and C. More complicated equations could be used to remove more than one row at a time from (17), but one should research appropriate texts for the equations.

IV. Calculating the Elements of the ABC+N Impedance Matrix

In order to calculate the impedances of (16) and (17), we need to review the impedance calculations of two current paths: The impedances associated with currents where all currents stay with the metallic phase or neutral conductors, and those inductances where current goes out on a metallic phase or neutral conductor and has an earth return.

A. Impedance for All Current in Overhead Metallic Conductors

This impedance refers to the case where current goes out on one metallic conductor and comes back on another and does not enter into the earth. The calculations are explained in many textbooks in the Power Engineering field (e.g., [1] through [7], though there are many others). This paper reviews some of the results of their analysis, but does not derive the equations. It might be noted that [7] is a hard to find resource, published in 1933, that is included because it shows the perspective of power system analysis in a foundational period and is an early user of what is commonly called Carson's equations (section IV.B).

The resistance of a line is relatively easily understood when viewed from a DC perspective, but AC resistance is more complicated than one might expect. For instance, [5] dedicates more than 20 pages and dozens of complex equations to the analysis and has only touched on the complications. Skin effect has a large impact on effective thickness of a conductor, raising R_{ac} with frequency. Eddy current losses increase effective resistance. Eddy current losses increase with frequency. Conductor stranding and core material (Fe vs. Cu vs. Al) affects eddy current losses. Magnetic steel cores have hysteresis losses. Each loss increases effective resistance. We will leave R_{ac} analysis to suggesting that one should use data provided by the wire manufacturers. These tables typically give effective AC resistance at 50 and/or 60 Hz and at a selection of temperatures.

One R_{ac} factor that can be adjusted for relatively accurately is temperature. If one needs to estimate R_{ac} at other temperatures than given in the tables, an equation that can be used, from [4], is

$$\frac{R_2}{R_1} = \frac{M + t_2}{M + t_1},$$

where

$M = 234.5$ for annealed 100% conductivity copper

$= 241.5$ for hard drawn 97.7% conductivity copper

$= 228.1$ for aluminum

t is in ^0C.

The referenced texts also derive the inductance calculations for overhead lines. We will not delve into the derivation deeply, but only review some important equations. One should make note that two assumptions of the calculations below are that the conductors are relatively remote from ground compared to their distance from each other, and that their distance from each other is large relative to the conductor radius. If the conductors are near the earth's semi-conductive media, or relatively close to one another, then current flow in the metallic paths and earth changes the flux distribution between the conductors or the current distribution in the conductor; hence, the equations of reactance given below start to have some error. We will proceed with the assumption that these are not important source of error.

Given spacing D_{21} from the center of conductor 1 to conductor 2 (where 1 and 2 represent any set of 2 conductors) and an effective conductor radius of r_{e1}, the reactance that is associated with the integration of flux from the center of conductor 1 out to conductor 2 is:

$$X_{21}^{self} = \frac{\omega\mu}{2\pi}\ln\left(\frac{D_{21}}{r_{e1}}\right)\ \Omega/\text{m} \quad . \qquad (19)$$

The "self" superscript is used to clarify that the reactance relates a voltage drop in a circuit for current in one of the wires in the circuit. In this case, conductor 1 is carrying current and we are calculating voltage drop in a loop involving conductor 1. The use or r_{e1} also tells us that this is a self inductance.

In the SI-MKS system, the exact numeric value of μ in free space is $4\pi 10^{-7}$, so the equation becomes

$$X_{21}^{self} = 2\cdot 10^{-7}\omega\ln\left(\frac{D_{21}}{r_{e1}}\right)\ \Omega/\text{m} \quad . \qquad (20)$$

A similar equation exists for mutual inductances. Given current in conductor 1, and spacing D_{21} and D_{31} from 1 to 2 and 1 to 3 respectively, the mutual reactance that reflects the voltage in the loop of conductor 2 and 3 by current in conductor 1 is:

$$\begin{aligned} X_{32}^{1,mutual} &= 2\cdot 10^{-7}\omega\left(\ln\left(\frac{D_{31}}{r_{e1}}\right) - \ln\left(\frac{D_{21}}{r_{e1}}\right)\right)\Omega/\text{m} \\ &= 2\cdot 10^{-7}\omega\ln\left(\frac{D_{31}}{D_{21}}\right)\ \Omega/\text{m} \end{aligned} \qquad (21)$$

If we consider current going out on conductor 1 and back on conductor 2, the reactance is doubled and (20) becomes

$$X_{12}^{\text{Total}} = 2\cdot 10^{-7}\omega\left(\ln\left(\frac{D_{12}}{r_{e2}}\right) + \ln\left(\frac{D_{21}}{r_{e1}}\right)\right)\ \ \Omega/\text{m} \qquad (22)$$

which can be restated as:

$$X_{12}^{\text{Total}} = 2\cdot 10^{-7}\omega\ln\left(\frac{D_{12}^{\ 2}}{r_{e1}r_{e2}}\right)\ \ \Omega/\text{m} \qquad (23)$$

or

$$X_{12}^{\text{Total}} = 4\cdot 10^{-7}\omega\ln\left(\frac{D_{12}}{\sqrt{r_{e1}r_{e2}}}\right)\ \Omega/\text{m} \quad . \qquad (24)$$

If $r_{e1} = r_{e2}$, this reduces to

$$X_{12}^{\text{Total}} = 4\cdot 10^{-7}\omega\ln\left(\frac{D_{12}}{r_e}\right)\ \Omega/\text{m} \quad . \qquad (25)$$

In (19) through (25), r_e is the conductor's effective radius for inductance calculations, not the actual radius. Some texts have an analysis that shows $r_e = 0.7788r_{actual}$, but this is under some simplified conditions that make the mathematics easier to work through. In real world applications, wire has an irregular surface shape, has internal discontinuities due to stranding, layering, and twisting, may have steel cores that modify the magnetic fields, and has current distribution modified by skin effects, proximity effects, and uneven conductivity from mixed levels of copper, aluminum, steel, and air gaps. Further, r_e will vary with frequency. Some texts have calculations that account for these effects to some degree, but in general it is likely more accurate to obtain the information for the wire in use from the wire manufacturer.

In the case of bundled conductors, texts show that rather than modeling each wire in the bundle, a sufficiently accurate scheme is to use an r_e calculated via the geometric mean radius (GMR) approach. The approach may be most easily understood via examples for a 2, 3, and 4 conductor bundle:

$$\begin{aligned} GMR_2 &= \left(r_{e1}\cdot r_{e2}\cdot D_{12}^2\right)^{1/4} \\ GMR_3 &= \left(r_{e1}\cdot r_{e2}\cdot r_{e3}\cdot D_{12}^2\cdot D_{13}^2\cdot D_{23}^2\right)^{1/9} \\ GMR_4 &= \left(r_{e1}\cdot r_{e2}\cdot r_{e3}\cdot r_{e4}\cdot D_{12}^2\cdot D_{13}^2\cdot D_{14}^2\cdot D_{23}^2\cdot D_{24}^2\cdot D_{34}^2\right)^{1/16} \end{aligned} \qquad (26)$$

Many texts show a derivation of positive and negative sequence impedance of regularly transposed lines that gives an impedance that is very similar to (20). The derivation assumes that all phase conductors have the same r_e, or in the case of bundled conductors, the same GMR as given in (26), and for unequal spacing between the conductors, that 3 transpositions are made that put each conductor in each position for 1/3 of the line length. The analysis of voltage drop for this canse effectively creates and derives a factor called the geometric mean distance (GMD) between the phase conductors. If phases A, B, and C, are distances D_{AB}, D_{AC}, and D_{CA} apart, GMD is

$$GMD = \left(D_{AB} \cdot D_{BC} \cdot D_{CA}\right)^{1/3} . \quad (27)$$

The positive and negative sequence impedances are then calculated as

$$X_1 = X_2 = \omega \cdot 2 \cdot 10^{-7} \cdot \ln\left(\frac{GMD}{GMR \text{ or } r_e}\right) \; \Omega/\text{m} . \quad (28)$$

Later in this paper a different approach to calculating sequence impedances is used that is appropriate for untransposed lines and that allows calculation the off diagonal elements in the sequence impedance matrix.

B. Impedance of the Earth Return Current Path

The exact path of current in the ground is difficult to analyze. There is likely some tendency to think of current entering the ground and distributing over a very wide area, somewhat as seen in Fig. 3, which would be appropriate for DC current flow. The current distribution for DC can be analyzed via a simple approach based on the inverse of path resistance; deep currents see a net higher path length; hence, higher resistance, so a proportionately lower current flows in that path.

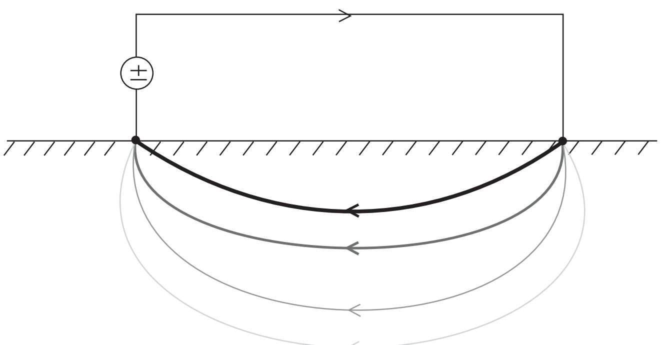

Fig. 3. Current Distribution for DC voltages

This deep penetration of current into the earth exists only for extremely low frequency and DC currents. For AC currents, something similar to conductor skin effect prevents current penetrating extremely deep into the earth. As flux penetrates the ground, it creates eddy currents that oppose the field (i.e., Lenz' law). These eddy currents ride along the surface; hence, currents tends to flow in the ground just underneath the conductor, as seen in Figs. 4 and 5.

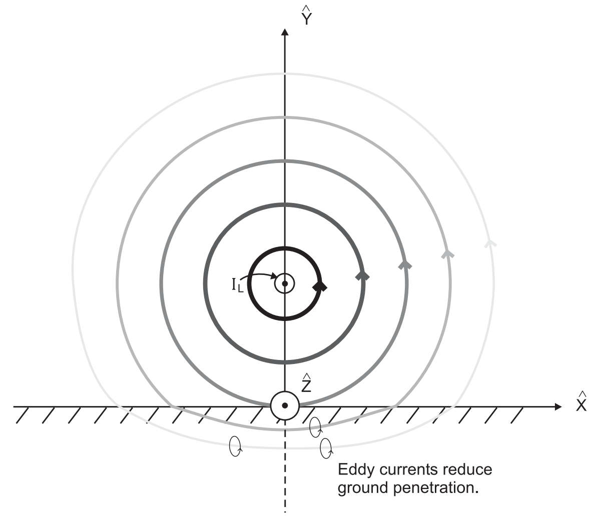

Fig. 4. H Field Due to Line Current

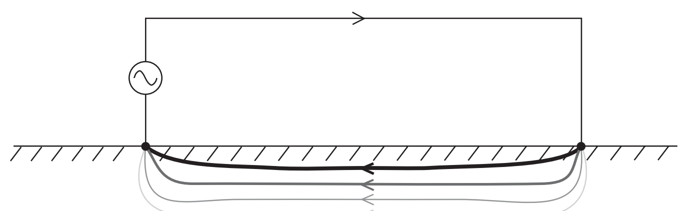

Fig. 5. Ground current distribution for AC voltages

This author has seen that analysis of net impedance for ground current flow is a complex problem that still has room for improvement even at this mature time in the power industry. A partial review of the methods for analyzing ground path impedances is [5] and [8]; the reviews give some hint to the broad spectrum and complexity of the topic. This author has not (yet) found a resource that starts from basic physics, explains the calculations in a detailed manner, works all the way to final calculations useful to the average engineer, and drops steps that lose the average person (at least this author).

The work of John Carson [9] is a common reference on impedance for current with ground return, and it appears to be the benchmark against which other analysis process are measured. However, it seems most use a "have faith it is correct" approach and have not studied his analysis in detail. It might be noted that [7], a text published in 1933, shortly after Carson wrote his paper, has heavy reliance upon Carson's analysis, but this text does not give a derivation of Carson's work, and the equations in [7] have been found in subsequent texts, such as [4].

Many resources write about of Carson's equations in terms of how to best analyze the infinite series equation that he presented as part of his analysis. However, this series was given as a means to analyze his equation for the self impedance of a wire with ground return. The equation, in SI/MKS terms (Carson's paper is presented using a variation of the cgs system) has the form of

$$Z = r_{wire} + \frac{j\omega\mu}{2\pi}\ln\left(p''/p_r\right) + \frac{\omega\mu}{\pi}\int_0^\infty \left(\left(\eta^2 + j\right)^{0.5} - \eta\right) e^{-2h'\eta} d\eta \ . \qquad (29)$$

The source of this equation is difficult to follow; Carson's development was very limited. A study of Carson's paper finds that it contains many equations placed on the page without development or proof, and this author has not been able to follow his logic in several places. Follow-up papers to Carson's work that develop, trace, prove, or expand upon his theory in good detail, and (29), have not been uncovered yet, though [9] does a good derivation of part of (29). There are likely obscure sources; this author would appreciate leads for these. The author has a German paper on the topic, but needs a translator.

However puzzling (29) may be, Carson's analysis and resultant equations appear to give satisfactory answers to ground path impedances, as based on the number of sources that state that they use his equations and are satisfied with the results. Given an acceptance of his equations, one could analyze (29) numerically, but the practice has been to utilize his infinite series to analyze the equation. Some references show that only the first few terms in his series are significant at the frequencies with which power systems are dealing. An equation provided in [2], [4], [5], and [7] that is reported as a good approximation of ground current impedance based on Carson's work where all but the first term in his series is dropped is

$$Z_{\#\#}^{GndLoop} = \left(r_{\#} + 0.00159 f\right) + j\left(0.004657 f \log_{10}\frac{2160\sqrt{\dfrac{\rho}{f}}}{\text{GMR (feet)}}\right)\Omega/\text{mile} \ , \qquad (30)$$

where

$\#\#$ refers to AA, BB, CC, or NN in (17)

$r_{\#}$ = resistance of conductor A, B, C, or N, in Ω/mile

f = frequency in cycles/sec

ρ = earth resistivity in Ω/meter^3 (though (30) is in feet, ρ is in still in meters^3)

GMR (or r_e) is the effective radius of the overhead conductor, in feet.

Restating (30) in units consistent with (24), (28), and other equations above (using ω rather than *f*, *m* rather than *ft* or *mi*, and ln rather than $\log_{10}$) the equation becomes

$$Z_{\#\#}^{GndLoop} = \left(r_{\#} + 1.571 \cdot 10^{-7}\omega\right) + j\left(2 \cdot 10^{-7}\omega \ln\frac{1650\sqrt{\dfrac{\rho}{\omega}}}{GMR \text{ or } r_e \text{ (in meters)}}\right)\Omega/\text{meter} \ . \qquad (31)$$

A concept discussed in some texts to help envision impedance of ground return current is drawn from a comparison of (31) to either (23) or (28). The concept envisions an equivalent depth of an imaginary return conductor where the distance from the overhead conductor to the imaginary underground conductor causes (23) or (28) to give the same reactance as (31). Difficulty in applying the equivalent depth of return concept to (23) causes confusion when defining the value of r_e of the conductor in the ground. Some apparent choices to use are 1 meter vs. 1 foot vs. r_e of the overhead conductor. However, the choice causes a problem to arise: each selection of r_e of the ground conductor gives a different equivalent depth of the return conductor. For instance, some texts use a value of 1, but with the same units (feet, meters, inches, cm...) as the overhead conductor. Depending on how the equations are arranged, the underground conductor might have a radius of 1 meter or 1 foot or 1 something else. Recall that the resultant reactance must have the same value as (31). As r_e changes, in order to have a constant X in (23), the effective D_{12} must vary with each choice of r_e; hence, the equivalent depth changes based on arbitrary factors in how the equations are set up.

Another approach to determine the equivalent depth of the return conductor is by comparing (31) to (28), not (23). The value of this process is that (28) does not contain r_e of the return conductor. The resultant equivalent depth of return where (28) and (31) give the same results is

$$D_e = GMD = 1650\sqrt{\frac{\rho}{\omega}} \quad meters \qquad (32)$$

from the overhead conductor. As an example, if ρ = 100 Ω/m^3 and ω = 377, then D_e = 849.8 meters.

However, (32) is yet one more example showing that how equations are set up affects the equivalent depth of the return current. This author has not found a good evaluation of the current distribution in the earth in the vertical direction, but the resources that have been found (e.g., [7]) tend to show that current in the ground is flowing over a wide area under the overhead conductor, indicating that if the current flow is to be modeled as a single circular conductor, a fairly large r_e may be appropriate.

There are other variations of equations to calculate ground current path impedances, such as a concept that uses impedances based upon a complex number for the depth of an equivalent underground conductor. As previously mentioned, one might read the material in [5] and [8] to get a basic understanding of these concepts, though these resources do not fully develop the concepts. In the balance of the paper we use (31) for impedance of the ground current path.

C. Calculation of the Elements of $\mathbf{Z_{ABCN}}$

We now have the tools to calculate the elements of the impedance matrix.

C.1: Diagonal Elements

The diagonal elements of (17), Z_{AA}, Z_{BB}, Z_{CC}, and Z_{NN} are taken directly from evaluation of (31) for each of the indicated ground loops. i.e.,

$$\begin{aligned} Z_{AA} &= Z_{AA}^{GndLoop} \\ Z_{BB} &= Z_{BB}^{GndLoop} \\ Z_{CC} &= Z_{CC}^{GndLoop} \\ Z_{NN} &= Z_{NN}^{GndLoop} \end{aligned} \tag{33}$$

Recall from (16) and (17) that there are several sub-components of these quantities. For instance

$$Z_{AA} = R_A + R_{AG} + X_{AG} + X_{AA} \ . \tag{34}$$

One can see from (31) that the resistive portion is the wire resistance R_A plus an additional quantity that we can associate with R_{AG}. Also, one can assume the reactances of Z_{AA} etc. are half due to X_{AG} and half due to X_{AA}. Thus, the sub components are

$$\begin{aligned} R_{AG} &= R_{AA}^{GndLoop} - R_A \\ R_{BG} &= R_{BB}^{GndLoop} - R_B \\ R_{CG} &= R_{CC}^{GndLoop} - R_C \\ R_{NG} &= R_{NN}^{GndLoop} - R_N \end{aligned} \tag{35}$$

$$\begin{aligned} X_{AA} &= X_{AG} = 0.5X_{AA}^{GndLoop} \\ X_{BB} &= X_{BG} = 0.5X_{BB}^{GndLoop} \\ X_{CC} &= X_{CG} = 0.5X_{CC}^{GndLoop} \\ X_{NN} &= X_{NG} = 0.5X_{NN}^{GndLoop} \end{aligned} \ . \tag{36}$$

C.2: Off Diagonal Elements

The off diagonals, using Z_{AB} as an example, have the components of

$$Z_{AB} = R_{AG} + X_{AG} + X_{AB} \tag{37}$$

The calculation of R_{AG} and X_{AG} is seen in (35) and (36). The calculation of X_{AB} requires an application of the upper form of (21). First, X_{BB}, from (36), relates the flux in loop B due to B phase current out to the ground imaginary ground conductor. Since the A and B loops share a common ground return in our model, the flux in Loop A due to B current is this same X_{BB} related flux minus the integration of flux from the B conductor to the A conductor, as see in (20). The net equations are shown below. The first line in the equation list shows the applicable equations numbers.

$$\begin{aligned} Z_{AB} &= R_{AG}(eq.35) + X_{AG}(eq.36) + \\ &\qquad \left(X_{BB}(eq.36) - X_{AB}^{self}(eq.20)\right) \\ Z_{AC} &= R_{AG} + X_{AG} + \left(X_{CC} - X_{AC}^{self}\right) \\ Z_{AN} &= R_{AG} + X_{AG} + \left(X_{NN} - X_{AN}^{self}\right) \\ Z_{BA} &= R_{BG} + X_{BG} + \left(X_{AA} - X_{BA}^{self}\right) \\ Z_{BC} &= R_{BG} + X_{BG} + \left(X_{CC} - X_{BC}^{self}\right) \\ Z_{BN} &= R_{BG} + X_{BG} + \left(X_{NN} - X_{BN}^{self}\right) \\ Z_{CA} &= R_{CG} + X_{CG} + \left(X_{AA} - X_{CA}^{self}\right) \\ Z_{CB} &= R_{CG} + X_{CG} + \left(X_{BB} - X_{CB}^{self}\right) \\ Z_{CN} &= R_{CG} + X_{CG} + \left(X_{NN} - X_{CN}^{self}\right) \\ Z_{NA} &= R_{NG} + X_{NG} + \left(X_{AA} - X_{NA}^{self}\right) \\ Z_{NB} &= R_{NG} + X_{NG} + \left(X_{BB} - X_{NB}^{self}\right) \\ Z_{NC} &= R_{NG} + X_{NG} + \left(X_{CC} - X_{NC}^{self}\right) \end{aligned} \tag{38}$$

IV. Representing $\mathbf{Z_{ABC}}$ as $\mathbf{Z_{012}}$ Sequence Impedances

We have finally reached the step where we will calculate Z_0, Z_1, and Z_2. An understanding of symmetrical component theory is necessary to proceed. A fairly concise review of symmetrical component theory is available in [11], from which the material below will borrow, but [1] -[7] are some very good resources, if one wishes a very complete review. Recall the ABC domain voltage drop equation that we first introduced in (12)

$$\begin{bmatrix} V_{A,S} \\ V_{B,S} \\ V_{C,S} \end{bmatrix} - \begin{bmatrix} V_{A,R} \\ V_{B,R} \\ V_{C,R} \end{bmatrix} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \ . \tag{39}$$

We wish to convert (39) to the 012 domain voltage drop equation. Recall also the matrices that are used in the conversion from ABC to 012 domain, and vice versa are

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \text{ and } \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \ . \tag{40}$$

In this case we will multiply (39) across by $\mathbf{A^{-1}}$. One should recall that, in matrix mathematics, if we multiply both sides of the equation by the same matrix, we have not have changed the equality of the two sides of the equation. Further, if we multiply by $\mathbf{A{\cdot}A^{-1}}$ (which calculates to be a unity matrix; i.e., 1's on the diagonal and 0's off the diagonal) we have effectively only multiplied by 1, so a valid modification of (39) is

$$\mathbf{A}^{-1}\begin{bmatrix}V_{A,S}\\V_{B,S}\\V_{C,S}\end{bmatrix}-\mathbf{A}^{-1}\begin{bmatrix}V_{A,R}\\V_{B,R}\\V_{C,R}\end{bmatrix}= \mathbf{A}^{-1}\begin{bmatrix}Z_{AA}&Z_{AB}&Z_{AC}\\Z_{BA}&Z_{BB}&Z_{BC}\\Z_{CA}&Z_{CB}&Z_{CC}\end{bmatrix}\mathbf{A}\cdot\mathbf{A}^{-1}\begin{bmatrix}I_A\\I_B\\I_C\end{bmatrix} . \tag{41}$$

This equation, when multiplied out, will become the 012 domain voltage drop equation. The reader should already recognize in (41) that $\mathbf{A}^{-1}\mathbf{V}_{\mathbf{ABC}}$ and $\mathbf{A}^{-1}\mathbf{I}_{\mathbf{ABC}}$ converts the voltages and currents to $\mathbf{V}_{\mathbf{012}}$ and $\mathbf{I}_{\mathbf{012}}$. Note also that the portion $\mathbf{A}^{-1}\mathbf{Z}_{\mathbf{ABC}}\mathbf{A}$ is converting the $\mathbf{Z}_{\mathbf{ABC}}$ domain impedances to the $\mathbf{Z}_{\mathbf{012}}$ domain and has the form of:

$$\mathbf{Z}_{012}=\mathbf{A}^{-1}\begin{bmatrix}Z_{AA}&Z_{AB}&Z_{AC}\\Z_{BA}&Z_{BB}&Z_{BC}\\Z_{CA}&Z_{CB}&Z_{CC}\end{bmatrix}\mathbf{A}=\begin{bmatrix}Z_{00}&Z_{01}&Z_{02}\\Z_{10}&Z_{11}&Z_{12}\\Z_{20}&Z_{21}&Z_{22}\end{bmatrix} \tag{42}$$

The set of equations implied by (42) is

$$\begin{aligned}
Z_{00}&=\left(Z_{AA}+Z_{BA}+Z_{CA}+Z_{AB}+Z_{BB}+Z_{CB}+Z_{AC}+Z_{BC}+Z_{CC}\right)/3\\
Z_{01}&=\left(Z_{AA}+Z_{BA}+Z_{CA}+a^2Z_{AB}+a^2Z_{BB}+a^2Z_{CB}+aZ_{AC}+aZ_{BC}+aZ_{CC}\right)/3\\
Z_{02}&=\left(Z_{AA}+Z_{BA}+Z_{CA}+aZ_{AB}+aZ_{BB}+aZ_{CB}+a^2Z_{AC}+a^2Z_{BC}+a^2Z_{CC}\right)/3\\
Z_{10}&=\left(Z_{AA}+aZ_{BA}+a^2Z_{CA}+Z_{AB}+aZ_{BB}+a^2Z_{CB}+Z_{AC}+aZ_{BC}+a^2Z_{CC}\right)/3\\
Z_{11}&=\left(Z_{AA}+aZ_{BA}+a^2Z_{CA}+a^2Z_{AB}+Z_{BB}+aZ_{CB}+aZ_{AC}+a^2Z_{BC}+Z_{CC}\right)/3\\
Z_{12}&=\left(Z_{AA}+aZ_{BA}+a^2Z_{CA}+aZ_{AB}+a^2Z_{BB}+Z_{CB}+a^2Z_{AC}+Z_{BC}+aZ_{CC}\right)/3\\
Z_{20}&=\left(Z_{AA}+a^2Z_{BA}+aZ_{CA}+Z_{AB}+a^2Z_{BB}+aZ_{CB}+Z_{AC}+a^2Z_{BC}+aZ_{CC}\right)/3\\
Z_{21}&=\left(Z_{AA}+a^2Z_{BA}+aZ_{CA}+a^2Z_{AB}+aZ_{BB}+Z_{CB}+aZ_{AC}+Z_{BC}+a^2Z_{CC}\right)/3\\
Z_{22}&=\left(Z_{AA}+a^2Z_{BA}+aZ_{CA}+aZ_{AB}+Z_{BB}+a^2Z_{CB}+a^2Z_{AC}+aZ_{BC}+Z_{CC}\right)/3
\end{aligned} \tag{43}$$

The form of the resultant 012 voltage drop equation is

$$\begin{bmatrix}V_{0,S}\\V_{1,S}\\V_{2,S}\end{bmatrix}-\begin{bmatrix}V_{0,R}\\V_{1,R}\\V_{2,R}\end{bmatrix}=\begin{bmatrix}Z_{00}&Z_{01}&Z_{02}\\Z_{10}&Z_{11}&Z_{12}\\Z_{20}&Z_{21}&Z_{22}\end{bmatrix}\begin{bmatrix}I_0\\I_1\\I_2\end{bmatrix}, \tag{44}$$

which can be abbreviated further to

$$\mathbf{V}_{012,\mathbf{S}}-\mathbf{V}_{012,\mathbf{R}}=\mathbf{Z}_{012}\cdot\mathbf{I}_{012} . \tag{45}$$

The definition of the $\mathbf{Z}_{\mathbf{012}}$ impedance matrix and its elements are

$$\mathbf{Z}_{012}=\begin{bmatrix}Z_{00}&Z_{01}&Z_{02}\\Z_{10}&Z_{11}&Z_{12}\\Z_{20}&Z_{21}&Z_{22}\end{bmatrix} \tag{46}$$

where:

$Z_{00} = (V_{0,S} - V_{0,R}) / I_0$
$Z_{10} = (V_{1,S} - V_{1,R}) / I_0$
$Z_{20} = (V_{2,S} - V_{2,R}) / I_0$
$Z_{01} = (V_{0,S} - V_{0,R}) / I_1$
$Z_{11} = (V_{1,S} - V_{1,R}) / I_1$
$Z_{21} = (V_{2,S} - V_{2,R}) / I_1$
$Z_{02} = (V_{0,S} - V_{0,R}) / I_2$
$Z_{12} = (V_{1,S} - V_{1,R}) / I_2$
$Z_{22} = (V_{2,S} - V_{2,R}) / I_2$.

Note we finally have reached a definition of $\mathbf{Z}_0$: The zero sequence impedance of the line will be Z_{00} in (46). Similarly, the positive sequence impedance will be Z_{11} and negative sequence impedance will be Z_{22}. This process of calculating $\mathbf{Z}_{\mathbf{012}}$ from the individual elements from the phase components is that we have a means of seeing the non-ideal nature of the line impedances; e.g., any off diagonal element in (46) causes cross coupling of the symmetrical component networks. We have a means of seeing the effect of non-transposition of lines.

In transmission lines, especially when phases are symmetrically spaced and phases are regularly transposed, it is frequently justified to assume the following symmetries in the $\mathbf{Z}_{\mathbf{ABC}}$ impedance network:

$$\begin{aligned}
&\mathrm{X_{SELF}}\left(X_S\right)=X_{AA}=X_{BB}=X_{CC}\\
&\mathrm{X_{MUTUAL}}\left(X_M\right)=X_{AB}=X_{AC}=X_{BA}=X_{BC}=X_{CA}=X_{CB}\\
&\mathrm{X_{GROUND}}\left(X_G\right)=X_{AG}=X_{BG}=X_{CG}\\
&\mathrm{R_{PHASE}}\left(R_P\right)=R_A=R_B=R_C\\
&R_{GROUND}\left(R_G\right)=R_{AG}=R_{BG}=R_{CG}
\end{aligned} \tag{47}$$

Substituting these impedances into (11) gives a relatively simple format for $\mathbf{Z}_{\mathbf{ABC}}$ with only two elements,

$$\mathbf{Z}_{\mathbf{ABC}}=\begin{bmatrix}Z_S&Z_M&Z_M\\Z_M&Z_S&Z_M\\Z_M&Z_M&Z_S\end{bmatrix} \tag{48}$$

where

$$\begin{aligned}
Z_S&=Z_{AA}=Z_{BB}=Z_{CC}\\
&=R_P+R_G+j\left(X_S+X_G\right)\\
Z_M&=Z_{AB}=Z_{AC}=Z_{BA}=Z_{BC}=Z_{CA}=Z_{CB}\\
&=R_G+j\left(X_M+X_G\right)
\end{aligned} . \tag{49}$$

When we convert this form of $\mathbf{Z}_{\mathbf{ABC}}$ to $\mathbf{Z}_{\mathbf{012}}$, we obtain the relatively simple diagonal matrix

$$\mathbf{Z_{012}} = \mathbf{A}^{-1}\begin{bmatrix} Z_S & Z_M & Z_M \\ Z_M & Z_S & Z_M \\ Z_S & Z_M & Z_S \end{bmatrix}\mathbf{A} = \begin{bmatrix} Z_S+2Z_M & 0 & 0 \\ 0 & Z_S-Z_M & 0 \\ 0 & 0 & Z_S-Z_M \end{bmatrix} \quad (50)$$

Hence, the 012 domain voltage drop equation becomes

$$\begin{bmatrix} V_{0,S} \\ V_{1,S} \\ V_{2,S} \end{bmatrix} - \begin{bmatrix} V_{0,R} \\ V_{1,R} \\ V_{2,R} \end{bmatrix} = \begin{bmatrix} Z_S+2Z_M & 0 & 0 \\ 0 & Z_S-Z_M & 0 \\ 0 & 0 & Z_S-Z_M \end{bmatrix}\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} . \quad (51)$$

Equation (51) is typically stated as

$$\begin{bmatrix} V_{0,S} \\ V_{1,S} \\ V_{2,S} \end{bmatrix} - \begin{bmatrix} V_{0,R} \\ V_{1,R} \\ V_{2,R} \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad (52)$$

where

$$\begin{aligned} Z_0 &= Z_S + 2Z_M \\ &= (R_P + 3R_G) + j(X_S + 2X_M + 3X_G) \\ Z_1 = Z_2 &= Z_S - Z_M \\ &= R_P + j(X_S - X_M) \end{aligned} \quad (53)$$

Also note that via some algebraic manipulation of (53)

$$\begin{aligned} Z_S &= (Z_0 + 2Z_1)/3 \\ Z_M &= (Z_0 - Z_1)/3 \end{aligned} \quad (54)$$

V. Zero Sequence Mutual Coupling Between Adjacent Power Lines

When current flows in a line, it creates flux that induces a voltage in any adjacent conductive loop. Assume Lines X and Y. Each conductor in X will be in different space relationship to each conductive loop in Y. There are many mechanisms for line Y to induce a voltage in X. For instance, assume a system with 5 conductors in each line X and Y, with 3 phases, 1 overhead ground wire N, and an assumed common ground conductor G. In Y there are 10 loops: AB, AC, AN, AG, BC, BN, BG, CN, CG, and NG. Next, the source line Y has 5 wires, each with different spacing relative to the 10 loops, so there is a net of 50 different coupling mechanisms between X and Y, and 50 more between Y and X. The number of coupling loops would be even higher if the system were modeled with multiple underground conductors.

The coupling mechanism that tends to receive most attention is zero sequence coupling. The A-G, B-G, C-G, and N-G loops tend to be large relative to the phase-phase and phase-N loops, as seen in Fig. 6; hence, they tend toward larger coupling. The coupling X and Y varies depending which phase in Y is carrying current; Phase C on X may be physically closer to line Y than Phase A, resulting in a higher coupling for current on phase C. So, when one speaks of zero sequence coupling, one is using a single lumped parameter to represents a complex coupling scenario.

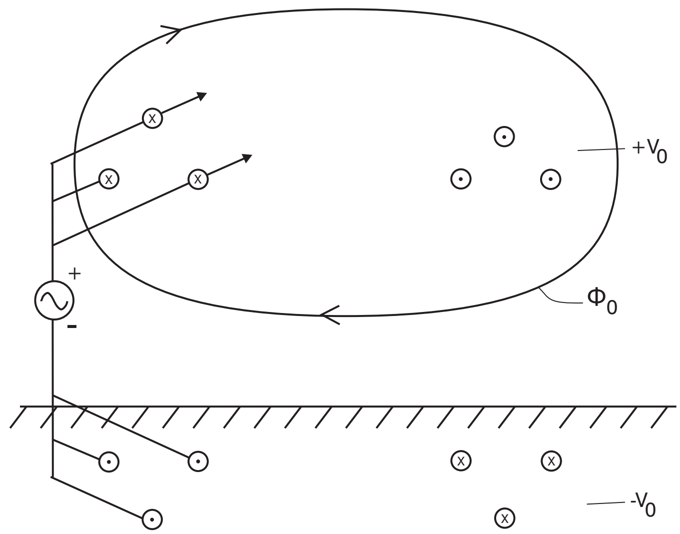

Fig. 6: Zero Sequence Coupling

The coupling between a line with current and a loop of known dimensions has the units of impedance: $Z_{Mutual}=V_{Induced}/I_{Source}$ For a given current and an adjacent loop, the equation for mutual impedance was previously given as (21). However, the difficulty in analysis of (21) for ground loops is that the effective location of the ground conductor is not well known, so it is difficult to determine the values for $D_{\#\#}$ in (21). For the analysis, once again, we will refer back to [4] and follow along with the industry tendency to have faith in Carson's work and find that others ([4], [7]) have published an approximation, based on Carson's work, of mutual coupling between two lines, of the form of

$$\begin{aligned} Z_{12}^{Mutual} &= \frac{V \text{ wire in ckt 1}}{I \text{ wire in ckt 2}} \\ &= 0.00159f + \\ & j\left(0.004657f\log_{10}\frac{2160\sqrt{\frac{\rho}{f}}}{D_{12}\text{ (feet)}}\right)\Omega/\text{mile} \end{aligned} \quad (55)$$

Note the high similarity to (30). Restating the equation in units of ω rather than *f*, *m* rather than *ft* or *mi*, and ln rather than $\log_{10}$, we obtain

$$Z_{12}^{\text{Mutual}} = 1.571 \cdot 10^{-7}\,\omega + j\left(2 \cdot 10^{-7}\,\omega \ln \frac{1650\sqrt{\dfrac{\rho}{\omega}}}{D_{12}}\right)\Omega/\text{meter} \quad (56)$$

To completely model zero sequence coupling between adjacent lines, one would apply this equation to the each wire pair between circuit 1 and 2. However, for simplicity, an average of the distances involved would give a basic approach.

VI. Summary, Conclusions

The previous material offers an approach to calculation of zero sequence impedances not directly seen in common texts. The approach starts by analyzing all impedances in the ABC domain. The paper shows how the presence of neutral conductors is included in the ABC domain impedance equations. After the ABC domain impedance matrix is calculated, the zero sequence and other sequence impedances are calculated using the equation $\mathbf{Z_{012}}=\mathbf{A}^{-1}\mathbf{Z_{ABC}}\mathbf{A}$, the conversion of ABC to 012 domain impedances sheds some light on off diagonal impedances in the Z_{012} domain impedances.

Work that the author wishes to address in the future is a true derivation of Z_{AA}, Z_{BB}, Z_{CC}, and Z_{NN} as well as the mutual impedances described in Section V. This paper relies upon the acceptance of others that the equations given for analysis of these quantities are sufficiently accurate. However, use of these equations involves a "have faith" element; therefore, the author cannot state their ultimate accuracy.

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Biography

John Horak received his BSEE degree from the University of Houston in 1988 and his MSEE degree, specializing in power system analysis, from the University of Colorado, Denver, in 1995. John joined Basler Electric in 1997 and is a Senior Application Engineer. Previous work included ten years with Stone and Webster Engineering where he was on assignment for six years in the System Protection Engineering offices of Public Service Company of Colorado. His work has included fault calculations, relay coordination settings, control design, and equipment troubleshooting. Other previous employers include Houston Lighting and Power and Chevron. He has authored, coauthored, and presented technical papers at a variety of industry events and conferences. John is a member of IEEE-IAS and -PES and has P.E. license in Colorado. (Contact: johnhorak@basler.com or john.horak@att.net)

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