A REVIEW OF NEGATIVE SEQUENCE CURRENT

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Introduction

Sequence component analysis plays an essential role in analyzing power system faults and explaining some power system phenomena. It is very well known that negative sequence current could cause rotor damage, and that damage is highly detrimental to rotating machines such as motors and generators.

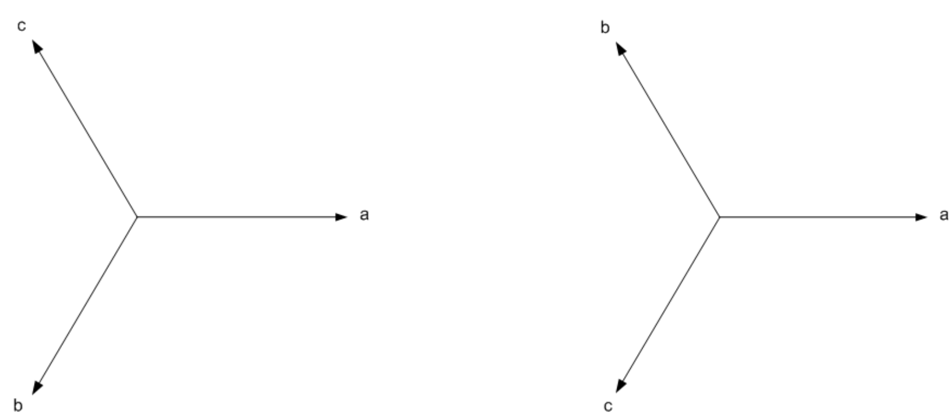
IEEE Tutorial of the Protection of Synchronous Generators (95 TP 102) has the following statement in section "Current Unbalance Protection": "During unbalanced conditions, negative sequence current is produced. The negative sequence current component rotates in the opposite direction from the rotor." This statement is not quite correct. Positive, negative and zero sequence currents are linear combinations of phase currents; thus, the vector of each sequence current rotates in the same direction as the phase current.

Usually, phase angle is measured with a reference, and the rotation of the negative sequence current generally is ignored. In *IEEE Standard for Synchrophasors for Power Systems* (C37.118-2005), the absolute phase angle is defined. It is worthwhile to clarify the rotation direction of the negative sequence current to avoid potential confusion in synchrophasor metering.

First, this paper reviews the concept of sequence components. Then, it explains that all sequence components rotate in the same direction. In a rotating machine, the negative sequence current vector rotates in the same direction as the rotor. It is the magnetic flux produced by the negative sequence current that rotates in the reverse direction of the rotor. Thus, the rotor cuts through the flux at twice the synchronous speed, and the induced current in the rotor is twice the line frequency.

I. Phase Rotation

In a power system, phase rotation is defined in the domain of a balanced three phase system. In a balanced system, phasors *a*, *b* and *c* are equal in magnitude and displaced 120° from each other. If phase *a* leads phase *b* by 120° and phase *b* leads phase *c* by 120°, this system is of *abc* rotation. If phase *a* lags phase *b* by 120° and phase *b* lags phase *c* by 120°, this system is of *acb* rotation. Phasor indication of phase rotation is illustrated in Figure 1.



abc rotation *acb* rotation

Figure 1. Phase Rotations

II. Concept of Sequence Components

Sequence component, also called symmetrical component, was introduced by C. L. Fortescue almost a century ago. In this section we will review the concept of sequence components [1][2].

2.1 Definition of Sequence Components

In the method of symmetrical components, three unbalanced phasors in phase *abc* rotation, we define

a) Positive sequence components consisting of three voltage phasors, designated by $V_a^{(1)}$, $V_b^{(1)}$, and $V_c^{(1)}$, which are equal in magnitude, displaced from each other by 120° in phase angle, and $V_a^{(1)}$ leads $V_b^{(1)}$ by 120°, $V_b^{(1)}$ leads $V_c^{(1)}$ by 120°.

b) Negative sequence components consisting of three voltage phasors, designated by $V_a^{(2)}$, $V_b^{(2)}$, and $V_c^{(2)}$, which are equal in magnitude, displaced from each other by 120° in phase angle, and $V_a^{(2)}$ lags $V_b^{(2)}$ by 120°, $V_b^{(2)}$ lags $V_c^{(2)}$ by 120°.

c) Zero sequence components consisting of three voltage phasors, designated by $V_a^{(0)}$, $V_b^{(0)}$, and $V_c^{(0)}$, which are equal in magnitude and with no phase displacement from each other.

Figure 2 illustrates the definition of sequence components graphically.

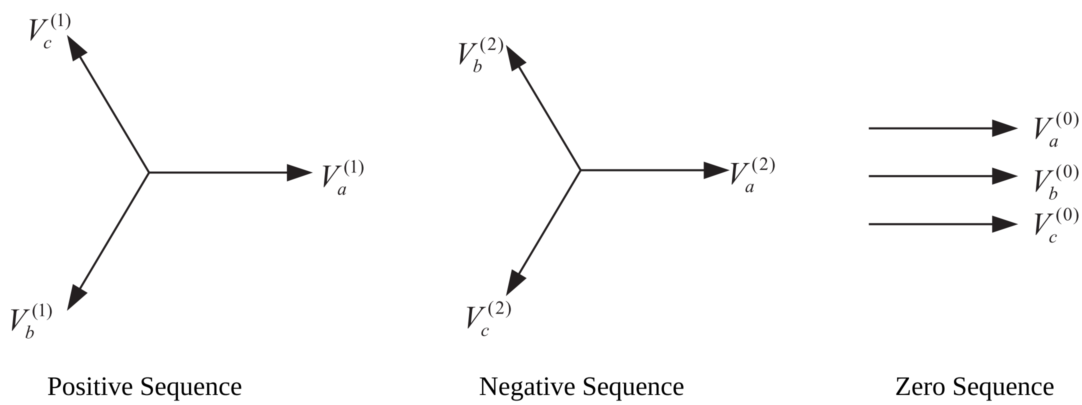


Figure 2. Definition of sequence component phasors

For a system with *acb* rotation, the definitions of positive sequence components and negative sequence components are swapped, that is,

a) Positive sequence components consisting of three voltage phasors, designated by $V_a^{(1)}$, $V_b^{(1)}$, and $V_c^{(1)}$, which are equal in magnitude, displaced from each other by 120° in phase angle, and $V_a^{(1)}$ lags $V_b^{(1)}$ by 120°, $V_b^{(1)}$ lags $V_c^{(1)}$ by 120°.

b) Negative sequence components consisting of three voltage phasors, designated by $V_a^{(2)}$, $V_b^{(2)}$, and $V_c^{(2)}$, which are equal in magnitude, displaced from each other by 120° in phase angle, and $V_a^{(2)}$ leads $V_b^{(2)}$ by 120°, $V_b^{(2)}$ leads $V_c^{(2)}$ by 120°.

The definition of the zero sequence components stays the same regardless whether the system has *abc* or *acb* phase rotation. If not specifically declared, a system is assumed to have phase *abc* rotation.

2.2 Computation of Sequence Components of Unbalanced Phasors

With the introduction of sequence components, we will show that any unbalanced set of three phase phasors can be decomposed into three sets of balanced sequence components, i.e., positive sequence components, negative sequence components and zero sequence components. Figure 3 shows three sets of balanced phasors which are the sequence component phasors of three unbalanced phasors.

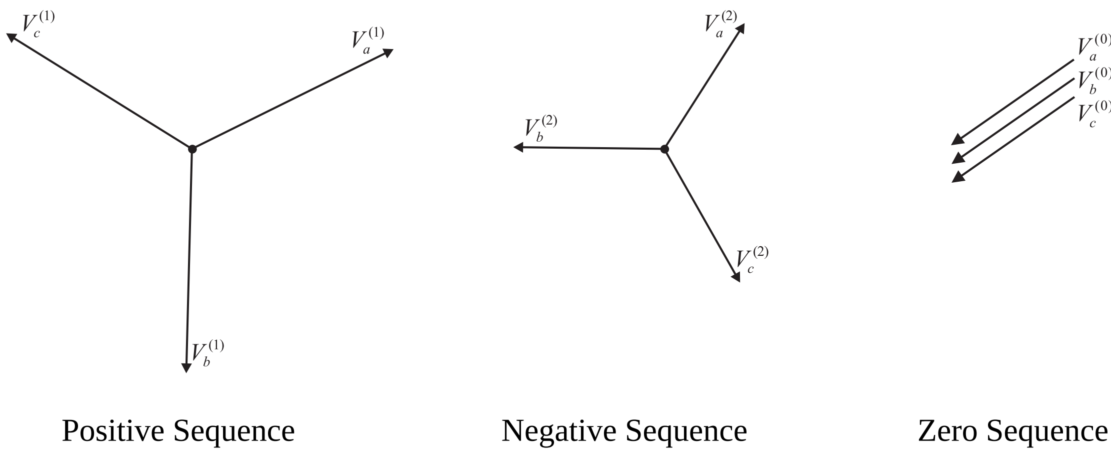


Figure 3. Sequence component phasors of three unbalanced phasors

Let us take an example of a set of unbalanced voltage phasors V_a, V_b, and V_c. We know the voltage phasor of each phase is the sum of its positive, negative and zero sequence components, i.e.,

$$V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)} \quad (1)$$
$$V_b = V_b^{(0)} + V_b^{(1)} + V_b^{(2)} \quad (2)$$
$$V_c = V_c^{(0)} + V_c^{(1)} + V_c^{(2)} \quad (3)$$

Figure 4 shows the graphical addition of the different sequence components for each phase.

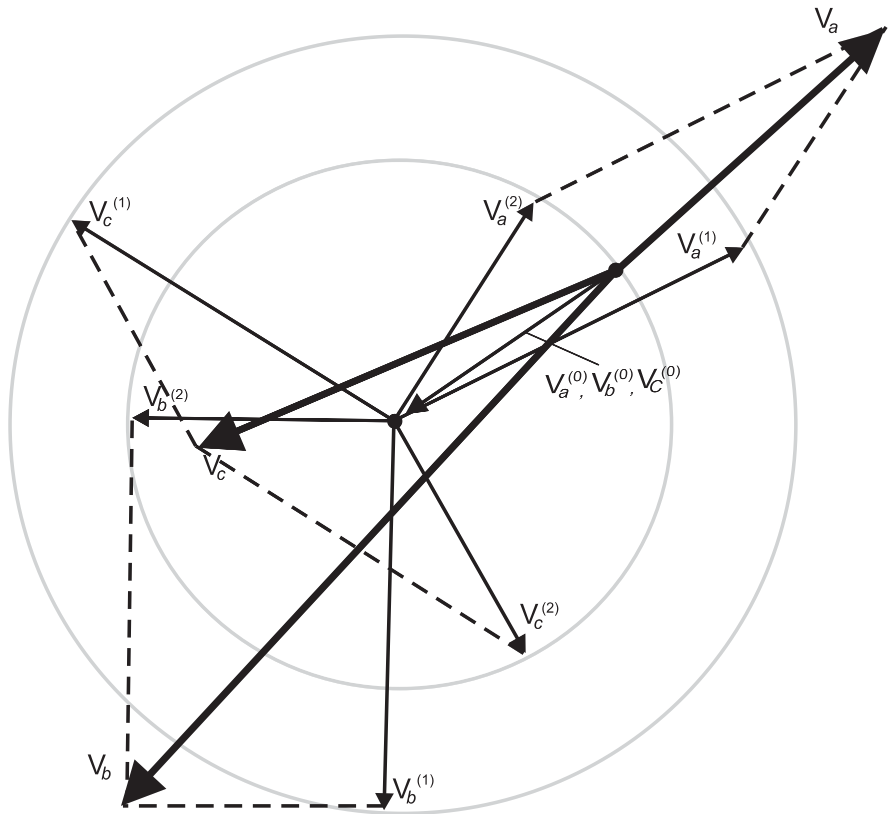


Figure 4. Graphical sum of different sequence components from each phase

It appears that we have nine unknowns in a system of three equations (1) ~ (3). However, there are only three independent variables. Let us choose $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$ as the three independent variables. From the definition of sequence components,

$$V_b^{(0)} = V_a^{(0)}, \qquad V_c^{(0)} = V_a^{(0)} \tag{4}$$

If we define an operator,

$$a = 1\angle 120° \tag{5}$$

then

$$a^2 = 1\angle 240°$$

We have the following relations from the definition of positive and negative sequence components,

$$V_b^{(1)} = a^2 V_a^{(1)}, \; V_c^{(1)} = a V_a^{(1)} \tag{6}$$
$$V_b^{(2)} = a V_a^{(2)}, \; V_c^{(2)} = a^2 V_a^{(2)} \tag{7}$$

Substituting equation (4), (6) and (7) into equation (2) and (3), a system of equations (1), (2) and (3) can be rewritten as,

$$V_a = V_a^{(0)} + \quad V_a^{(1)} + \quad V_a^{(2)} \tag{8}$$

$$V_b = V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} \tag{9}$$

$$V_c = V_a^{(0)} + a\, V_a^{(1)} + a^2 V_a^{(2)} \tag{10}$$

Now we have three unknowns $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$ with three equations. Solving this system of linear equations, we have

$$V_a^{(0)} = \frac{1}{3}(V_a + V_b + V_c) \tag{11}$$

$$V_a^{(1)} = \frac{1}{3}(V_a + aV_b + a^2 V_c) \tag{12}$$

$$V_a^{(2)} = \frac{1}{3}(V_a + a^2 V_b + aV_c) \tag{13}$$

Let take an example of an umbalanced system with phase ground voltages $V_a = 53\angle 43.5°\ V$, $V_b = 107\angle 229.5°\ V$ and $V_c = 67\angle 205.5°\ V$. Substituting these values and equation (5) into equation (11) ~ (13), we get $V_a^{(0)} = 39.17\angle -141.15°\ V$, $V_a^{(1)} = 56.95\angle 29.36°\ V$ and $V_a^{(2)} = 38.35\angle 59.76°\ V$.

Equations (11), (12) and (13) give not only a mathematical method to calculate sequence components but also a graphical method to derive sequence component phasors. The zero sequence components are one third of the sum of the unbalanced vectors.

From equation (11), the zero sequence components are one third of the vector sum of three phase vectors. Figure 5 illustrates the derivation of phase *a* zero sequence component.

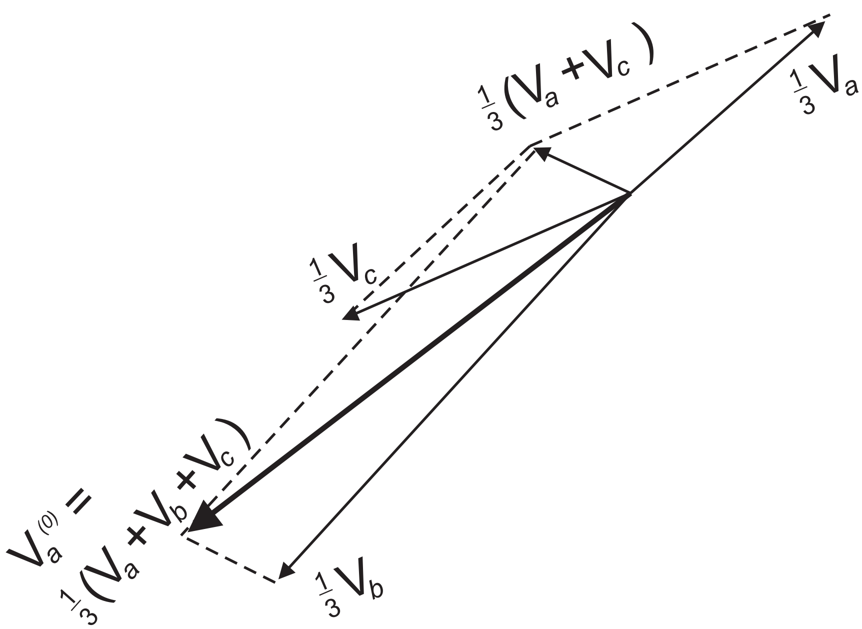


Figure 5. Derivation of phase *a* zero sequence component

Equation (12) displays a graphical method to build the positive sequence component of phase *a*. The operator *a* multiplying a vector is equivalent to rotating the vector by 120° counterclockwise. The operator a^2 multiplying a vector is equivalent to rotate the vector by 240° counterclockwise, or 120° clockwise. To derive phase *a* positive sequence component, V_b needs to be rotated by 120° counter-clockwise, and V_c needs to be rotated by 120° clockwise. Then the shifted vectors and V_a are added together and divided by 3. Figure 6 illustrates the derivation of phase *a* positive sequence component.

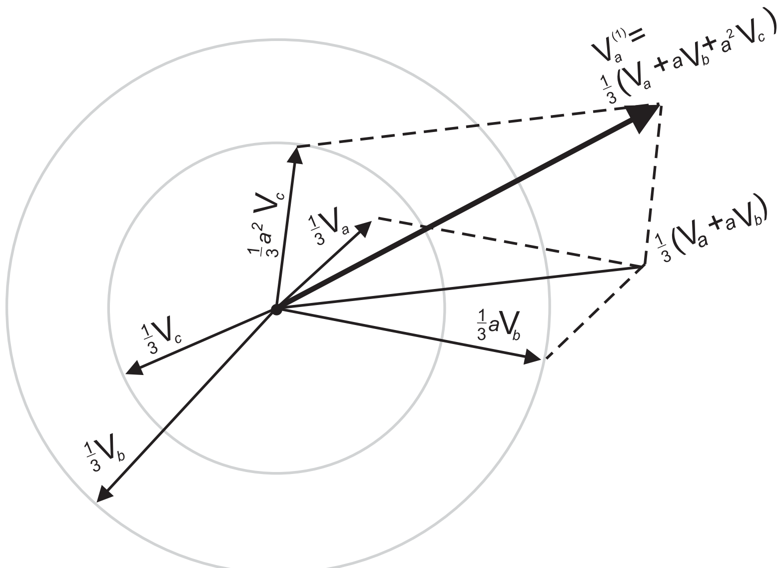


Figure 6. Derivation of phase *a* positive sequence component

Similarly, to derive phase a positive sequence component, V_b needs to be shifted 120° clockwise and V_c needs to be shifted 120° counterclockwise. Then the shifted vectors and V_a are added together and divided by 3. Figure 7 illustrates the derivation of phase *a* negative sequence component.

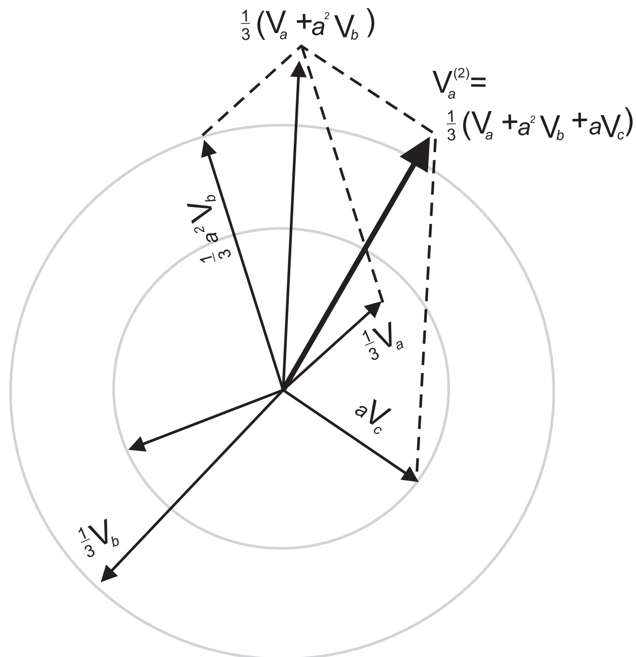


Figure 7. Derivation of phase *a* negative sequence component

With $V_a^{(0)}, V_a^{(1)}$, and $V_a^{(2)}$, sequence components for phase b and c can be computed easily by equations (4), (6), and (7).

2.3 Rotation of Sequence Components

In a rotating machine such as a generator or machine, phase voltages and current rotate with the rotor whether or not the system is balanced. In normal operating condition, the system is balanced or nearly balanced and positive sequence components dominate. When the system is unbalanced, a significant amount of negative sequence and/or zero sequence components could exist. There could be some confusion about the rotation of the negative sequence components. *IEEE Tutorial of the Protection of Synchronous Generators* (95 TP 102) has the following statement in section “Current Unbalance Protection”: “During unbalanced conditions, negative sequence current is produced. The negative sequence current component rotates in the opposite direction from the rotor. The flux produced by this current as seen by the rotor has a frequency of twice synchronous speed as a result of the reverse rotation combined with the positive rotation of the rotor”[3]. This statement may give us a false impression that positive sequence component rotates in the same direction with the rotor while the negative sequence component rotates in the opposite direction from the rotor.

However, from Figure 4, it appears that negative sequence components cannot possibly rotate in the different direction from the positive or zero sequence components. From equations (11) ~ (13), all sequence components are linear combinations of the three phase phasors. This tells us that the positive, negative and zero sequence currents all rotate in the same direction as the three phase phasors.

It is well-known that the negative sequence current can cause rapid heating and has more damaging effect on the rotor. To fully understand the effect of the negative sequence current, we need to understand the relationship between current and magnetic flux.

III. Analysis of Magnetics in a rotating machine

When current flows in a conductor, it creates a magnetic field. The intensity of the magnetic field is proportional to the amperes of the current. The relationship between the magnetic field and flux density is given by

$$\vec{B} = \mu \vec{H} \tag{14}$$

where μ is the permeability of the material. Since μ is a scalar constant, the vector $\vec{B}$ is in line with the vector $\vec{H}$. The direction of $\vec{B}$ or $\vec{H}$ is given by Right Hand Grip Rule. In Figure 8, current *I* flowing through a conductor in the direction indicated by the white arrow produces a magnetic flux field $\vec{B}$ around the conductor as shown by the arrows. The relationship between the current *I* and its induced flux density $\vec{B}$ at distance *r* from the conductor is

$$B = \frac{\mu}{2\pi r} I \tag{15}$$

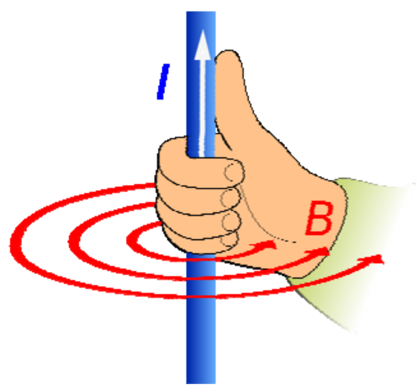


Figure 8. Right hand grip rule (from Wikipedia)

Let us consider a simple three phase, two pole machine. The stator contains a set of coils, each spaced 120° apart, as is shown in Figure 9.

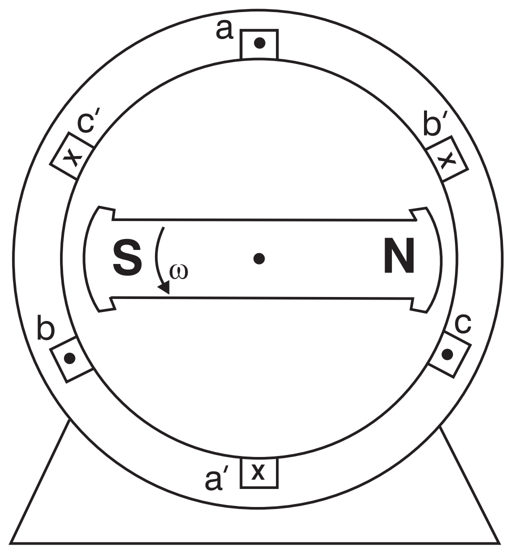


Figure 9. Simple three phase, two pole armature winding

In Figure 9, a circle in each slot represents part of a stator winding. A dot means the current flowing out of and a cross means the current flowing into the paper. Phase *a, b,* and *c* has stator windings aa', bb' and cc' respectively.

When the three-phase is unbalanced, the three-phase of unbalanced currents can be decomposed into three sets of balanced three-phase currents, i.e., zero sequence, positive sequence and negative sequence components. We will study the flux induced by each sequence current in the following sections. In sections 3.1 ~ 3.3, we will first derive the flux at the central point of the rotor. In section 3.4, we will have a case study of the flux induced by each sequence current at a point other than the center of the rotor.

3.1 Flux Induced by Zero Sequence Current

Zero sequence currents, by definition, are equal in magnitude and angle and can be expressed by

$$\begin{aligned} i_{0aa'} &= I_0 \sin(\omega t) \\ i_{0bb'} &= I_0 \sin(\omega t) \\ i_{0cc'} &= I_0 \sin(\omega t) \end{aligned} \tag{16}$$

Using the right hand grip rule, the directions of the induced flux density vectors are as indicated in Figure 10. Mathematically, the resulting magnetic flux density vectors are

$$\vec{B}_{0aa'} = B_0 \sin(\omega t)\angle 0° = B_0 \sin(\omega t) \tag{17}$$

$$\vec{B}_{0bb'} = B_0 \sin(\omega t)\angle 120° = B_0 \sin(\omega t)e^{j120°} \tag{18}$$

$$\vec{B}_{0cc'} = B_0 \sin(\omega t)\angle -120° = B_0 \sin(\omega t)e^{-j120°} \tag{19}$$

where $B_0 = \dfrac{\mu}{2\pi r} I_0$, *r* is the radius of the stator.

Since the windings aa', bb' and cc' are spaced 120° from each other, the directions of the induced fluxes are illustrated in Figure 10. The total flux induced by zero sequence currents is the sum of the flux from each phase of the zero sequence current. Since the zero sequence currents of phase *a*, *b* and *c* are exactly the same, the induced fluxes $\vec{B}_{0aa'}$, $\vec{B}_{0bb'}$ and $\vec{B}_{0cc'}$ are balanced and thus the net flux induced by zero sequence currents is zero.

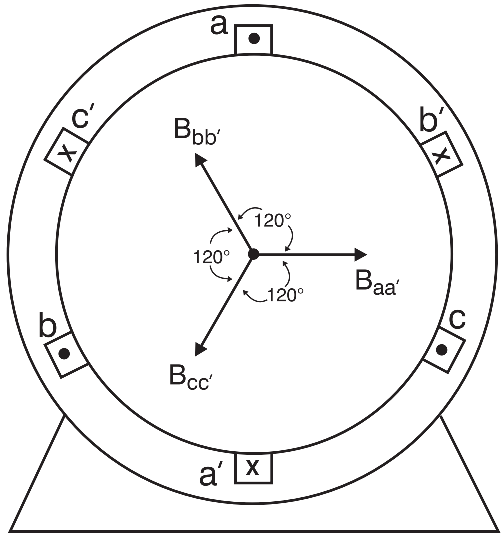


Figure 10. Directions of induced flux per assumed currents

Mathematically, we have

$$\vec{B}_0 = \vec{B}_{0aa'} + \vec{B}_{0bb'} + \vec{B}_{0cc'}$$

$$= B_0 \sin(\omega t) + B_0 \sin(\omega t)e^{j120^\circ} + B_0 \sin(\omega t)e^{-j120^\circ} \quad (20)$$

$$= B_0 \sin(\omega t)(1 + e^{j120^\circ} + e^{-j120^\circ})$$

$$= 0$$

This proves that no magnetic flux will be introduced in the rotor by zero sequence current in the stator.

3.2 Flux Induced by Positive Sequence Current

Positive sequence components, by its definition, can be expressed as

$$\begin{aligned} i_{1aa'} &= I_1 \sin(\omega t) \\ i_{1bb'} &= I_1 \sin(\omega t - 120^\circ) \\ i_{1cc'} &= I_1 \sin(\omega t + 120^\circ) \end{aligned} \quad (21)$$

If we keep the assumed direction of currents unchanged, the direction of the induced magnetic flux density vectors stays the same as that illustrated in Figure 10. For positive sequence currents, since the instantaneous magnitude of each phase is different, the net flux induced is not zero. Similar to the case of zero sequence currents, the flux density induced by the positive sequence currents can be expressed by

$$\vec{B}_{1aa'} = B_1 \sin(\omega t)\angle 0^\circ = B_1 \sin(\omega t) \quad (22)$$

$$\vec{B}_{1bb'} = B_1 \sin(\omega t - 120°)\angle 120° = B_1 \sin(\omega t - 120°)e^{j120°} \quad (23)$$

$$\vec{B}_{1cc'} = B_1 \sin(\omega t + 120°)\angle -120° = B_1 \sin(\omega t + 120°)e^{-j120°} \quad (24)$$

where $B_1 = \dfrac{\mu}{2\pi r} I_1$, r is the radius of the stator.

The sum of the three magnetic flux density vectors is

$$\vec{B}_1 = \vec{B}_{1aa'} + \vec{B}_{1bb'} + \vec{B}_{1cc'}$$

$$= B_1 \sin(\omega t) + B_1 \sin(\omega t - 120°)e^{j120°} + B_1 \sin(\omega t + 120°)e^{-j120°}$$

$$= B_1 \sin(\omega t) + B_1(\sin(\omega t)\cos 120° - \cos(\omega t)\sin 120°)e^{j120°} + B_1(\sin(\omega t)\cos 120° + \cos(\omega t)\sin 120°)e^{-j120°}$$

$$= B_1 \sin(\omega t) + B_1 \sin(\omega t)\cos 120°(e^{j120°} + e^{-j120°}) - B_1 \cos(\omega t)\sin 120°(e^{j120°} - e^{-j120°})$$

$$= B_1 \sin(\omega t) + B_1 \sin(\omega t)\cos(120°)(2\cos(120°)) - B_1 \cos(\omega t)\sin(120°)(2j\sin 120°)$$

$$= \frac{3}{2} B_1(\sin(\omega t) - j\cos(\omega t)) = \frac{3}{2} B_1(-j^2 \sin(\omega t) - j\cos(\omega t))$$

$$= -j\frac{3}{2} B_1(\cos(\omega t) + j\sin(\omega t))$$

$$= -j\frac{3}{2} B_1 e^{j\omega t} \quad (25)$$

From equation (25), we see that the amplitude of the magnetic flux density vector induced by positive sequence currents is a constant. Furthermore, the induced flux density vector rotates counterclockwise at angular velocity ω. Figure 11 (a), (b), (c) and (d) illustrate the net flux density induced by positive sequence currents at $\omega t = 0°, 90°, 180°$ and $270°$ respectively. In Figure 9, we see the rotor also rotates counterclockwise at angular velocity ω. Since there is no relative movement between the magnetic flux and the rotor, no current is induced in the rotor by the magnetic flux density. This shows that balanced three phase current, or positive sequence current, will not cause a rotor heating problem in the central part of the rotor.

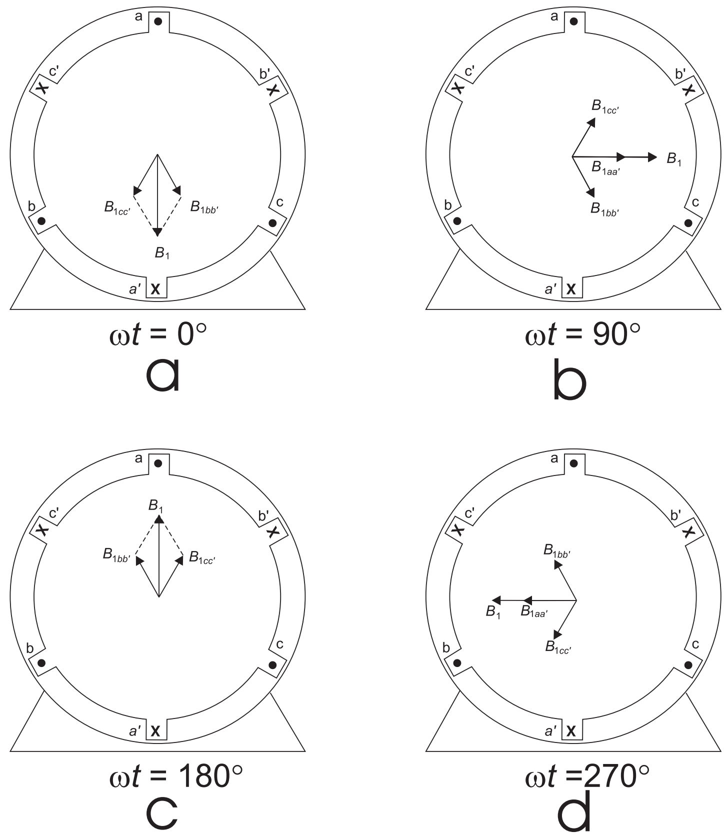


Figure 11. Flux induced by positive sequence currents rotates anticlockwise

3.3 Flux Induced by Negative Sequence Current

For negative sequence components, the analysis is similar to that of the positive sequence. The three phase negative sequence current can be expressed as

$$\begin{aligned} i_{2aa'} &= I_2 \sin(\omega t) \\ i_{2bb'} &= I_2 \sin(\omega t + 120°) \\ i_{2cc'} &= I_2 \sin(\omega t - 120°) \end{aligned} \tag{26}$$

The direction of the induced magnetic flux density vectors are the same as that from zero and positive sequence cases, as illustrated in Figure 10. Mathematically,

$$\vec{B}_{2aa'} = B_2 \sin(\omega t) \tag{27}$$

$$\vec{B}_{2bb'} = B_2 \sin(\omega t + 120°) e^{j120°} \tag{28}$$

$$\vec{B}_{2cc'} = B_2 \sin(\omega t - 120°) e^{-j120°} \tag{29}$$

where $B_2 = \dfrac{\mu}{2\pi r} I_2$, r is the radius of the stator.

The sum of the three magnetic flux density vectors is

$$\vec{B}_2 = \vec{B}_{2aa'} + \vec{B}_{2bb'} + \vec{B}_{2cc'}$$

$$= B_2 \sin(\omega t) + B_2 \sin(\omega t + 120°)e^{j120°} + B_2 \sin(\omega t - 120°)e^{-j120°}$$

$$= B_2 \sin(\omega t) + B_2 (\sin(\omega t)\cos 120° + \cos(\omega t)\sin 120°)e^{j120°} +$$
$$B_2 (\sin(\omega t)\cos 120° - \cos(\omega t)\sin 120°)e^{-j120°}$$

$$= B_2 \sin(\omega t) + B_2 \sin(\omega t)\cos 120°(e^{j120°} + e^{-j120°}) + B_2 \cos(\omega t)\sin 120°(e^{j120°} - e^{-j120°})$$

$$= B_2 \sin(\omega t) + B_2 \sin(\omega t)\cos(120°)(2\cos(120°)) + B_2 \cos(\omega t)\sin 120°(2j\sin 120°)$$

$$= \frac{3}{2} B_2 (\sin(\omega t) + j\cos(\omega t))$$

$$= -j\frac{3}{2} B_2 e^{-j\omega t} \qquad (30)$$

From equation (30), we see that the magnetic flux density vector induced by negative sequence current rotates clockwise at angular velocity *ω*. Figure 12 (a), (b), (c) and (d) illustrate the net flux density induced by negative sequence currents at $\omega t = 0°, 90°, 180°$ and $270°$ respectively. Since the rotor rotates counterclockwise at angular velocity *ω*, the magnetic flux density vector rotates with angular velocity *2ω* counterclockwise relative to the rotor.

The current induced at the rotor center is of double frequency. The induced current could cause rapid heating in the rotor, which may result in insulation failure and/or mechanical problems.

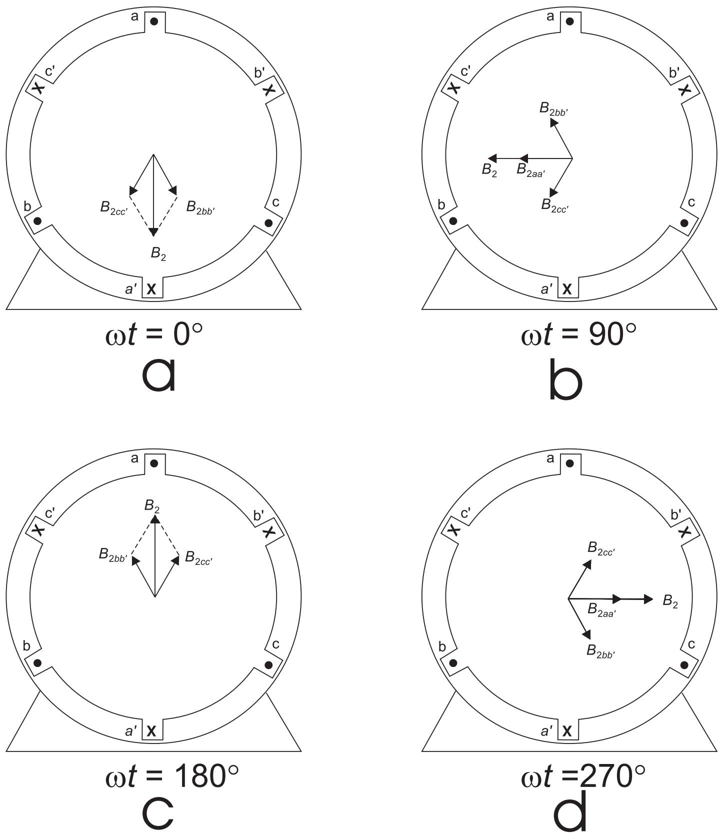


Figure 12. Flux induced by negative sequence currents rotates clockwise

3.4 Flux at a general point in the rotor

We have analyzed the flux density at the rotor center. The conclusion about the flux induced at the rotor center may not be generalized to any point in the rotor without proof. It is mathematically complicated, and a general mathematical formula may not give intuition about the relationship between the flux and each sequence component currents in the stator. In this section, we will mathematically analyze and numerically evaluate the flux density at a point halfway between the center and stator phase *a* current.

In Figure 13, point *A* is halfway between the rotor center and phase *a* conductor. Six induced flux density vectors are illustrated in Figure 13 per the right hand grip rule. Since $\angle DAE + \angle EAC = 90°$ and $\angle ACE + \angle EAC = 90°$, we have $\angle\alpha = \angle ECA$.

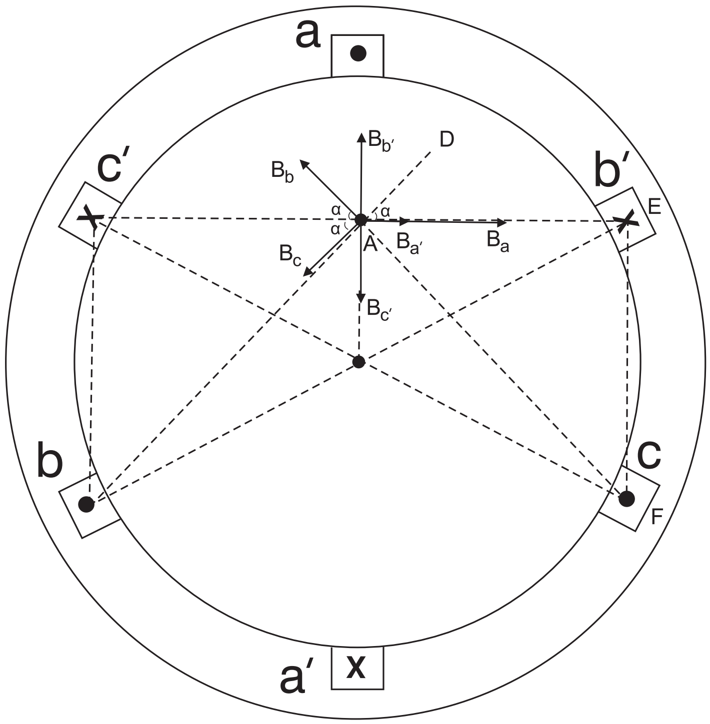


Figure 13. Flux density at half way between the center and phase *a* conductor

By doing some more geometry and trigonometry analysis, we have

$$\cos\alpha = \frac{2}{7}\sqrt{7}$$
$$\sin\alpha = \frac{1}{7}\sqrt{21} \quad (31)$$

The relationship between the current and its induced flux density is given by equation (15). Since we are only interested in the comparison of the impact of various sequence component currents, the factor $\frac{\mu}{2\pi}$ is dropped and we simply use $B = \frac{I}{r}$ in the following analysis.

The x and y components of the flux density vector induced by each phase of the stator sequence currents can be expressed as follows,

$$B_{ax} = 2\sin(\omega t) \quad (32)$$

$$B_{ay} = 0 \quad (33)$$

$$B_{apx} = \frac{2}{3}\sin(\omega t) \quad (34)$$

$$B_{apy} = 0 \quad (35)$$

$$B_{bx} = -\frac{2}{\sqrt{7}}\sin(\omega t - \theta)\cos\alpha \quad (36)$$

$$B_{by} = \frac{2}{\sqrt{7}}\sin(\omega t - \theta)\sin\alpha \tag{37}$$

$$B_{bpx} = 0 \tag{38}$$

$$B_{bpy} = \frac{2}{\sqrt{3}}\sin(\omega t - \theta) \tag{39}$$

$$B_{cx} = -\frac{2}{\sqrt{7}}\sin(\omega t + \theta)\cos\alpha \tag{40}$$

$$B_{cy} = -\frac{2}{\sqrt{7}}\sin(\omega t + \theta)\sin\alpha \tag{41}$$

$$B_{cpx} = 0 \tag{42}$$

$$B_{cpy} = -\frac{2}{\sqrt{3}}\sin(\omega t + \theta) \tag{43}$$

The above equations work for zero sequence, positive sequence and negative sequence currents with proper θ specified, i.e.,

$$\begin{aligned} &\theta = 0°, \text{ for zero sequence components} \\ &\theta = 120°, \text{ for positive sequence components} \\ &\theta = -120°, \text{ for negative sequence components} \end{aligned} \tag{44}$$

With the flux density components from each stator sequence currents, we can the x and y components of the total induced flux components at point *A*,

$$\begin{aligned} B_x &= B_{ax} + B_{apx} + B_{bx} + B_{bpx} + B_{cx} + B_{cpx} \\ &= 2\sin(\omega t) + \frac{2}{3}\sin(\omega t) - \frac{2}{\sqrt{7}}(\sin(\omega t - \theta)\cos\alpha - \frac{2}{\sqrt{7}}\sin(\omega t + \theta)\cos\alpha \\ &= \frac{8}{3}\sin(\omega t) - \frac{2}{\sqrt{7}}(\sin(\omega t - \theta) + \sin(\omega t + \theta))\cos\alpha \\ &= \frac{8}{3}\sin(\omega t) - \frac{4}{\sqrt{7}}\sin(\omega t)\cos\theta\cos\alpha \\ &= (\frac{8}{3} - \frac{4}{\sqrt{7}}\cos\theta\cos\alpha)\sin(\omega t) \end{aligned} \tag{45}$$

$$
\begin{aligned}
B_y &= B_{ay} + B_{apy} + B_{by} + B_{bpy} + B_{cy} + B_{cpy} \\
&= \frac{2}{\sqrt{7}}\sin(\omega t - \theta)\sin\alpha + \frac{2}{\sqrt{3}}\sin(\omega t - \theta) - \frac{2}{\sqrt{7}}\sin(\omega t + \theta)\sin\alpha - \frac{2}{\sqrt{3}}\sin(\omega t + \theta) \\
&= \frac{2}{\sqrt{7}}\sin\alpha(\sin(\omega t - \theta) - \sin(\omega t + \theta)) + \frac{2}{\sqrt{3}}(\sin(\omega t - \theta) - \sin(\omega t + \theta)) \\
&= -\frac{4}{\sqrt{7}}\sin\alpha\sin\theta\cos(\omega t) - \frac{4}{\sqrt{3}}\sin\theta\cos(\omega t) \\
&= -(\frac{4}{\sqrt{3}} + \frac{4}{\sqrt{7}}\sin\alpha)\sin\theta\cos(\omega t)
\end{aligned} \tag{46}
$$

$$
\begin{aligned}
\vec{B} &= B_x + jB_y \\
&= (\frac{8}{3} - \frac{4}{\sqrt{7}}\cos\theta\cos\alpha)\sin(\omega t) + j[-(\frac{4}{\sqrt{3}} + \frac{4}{\sqrt{7}}\sin\alpha)\sin\theta]\cos(\omega t)
\end{aligned} \tag{47}
$$

Substituting equation (31) and (44) into equation (47) and do some mathematical manipulations,, we have the flux density vectors for positive, negative and zero sequence currents respectively in equation (48), (49) and (50).

$$\vec{B}_1 = 3.2381\sin(\omega t) - j2.8571\cos(\omega t) = -j3.2381e^{j\omega t} + j0.381\cos\omega t \tag{48}$$

$$\vec{B}_2 = 3.2381\sin(\omega t) + j2.8571\cos(\omega t) = j3.2381e^{-j\omega t} - j0.381\cos\omega t \tag{49}$$

$$\vec{B}_0 = 1.5238\sin(\omega t) \tag{50}$$

From the above equations, it is apparent that the flux density vector at point *A* does not rotate at a constant angular speed ω. We can also see that $-j3.2381e^{j\omega t}$ is the dominant component of $\vec{B}_1$ and $j3.2381e^{-j\omega t}$ is the dominant component of $\vec{B}_2$. This tells us that the flux induced by the positive sequence currents rotates in the same direction of the rotor and the flux induced by the negative sequence currents rotates in the reverse direction of the rotor. Using equation (48) ~ (50), we can compute the flux density for any ωt for each sequence component currents. Table I lists the induced flux density by positive, negative and zero sequence component currents at various points of ωt. Note that the magnitude of the flux density is of relative importance only, assuming that each sequence current has the same magnitude.

Table I. Flux density vector at various current angles

	Positive sequence		Negative sequence		Zero sequence	
ωt	|B1|	|B1| angle	|B2|	|B2| angle	|B0|	|B0| angle
0°	2.86	270.00°	2.86	90.00°	0.00	0°
30°	2.96	303.20°	2.96	56.80°	0.76	0°
60°	3.15	333.00°	3.15	27.00°	1.32	0°
90°	3.24	0.00°	3.24	0.00°	1.52	0°
120°	3.14	27.00°	3.14	333.00°	1.32	0°
150°	2.96	56.80°	2.96	303.20°	0.76	0°
180°	2.86	90.00°	2.86	270.00°	0.00	180°
210°	2.96	123.20°	2.96	236.80°	0.76	180°
240°	3.15	153.00°	3.15	207.00°	1.32	180°
270°	3.24	180.00°	3.24	180.00°	1.52	180°
300°	3.15	207.00°	3.15	153.00°	1.32	180°
330°	2.96	236.80°	2.96	123.20°	0.76	180°

Table I verifies that the flux at a general point inside the rotor behaves not exactly the same compared to that at the center of the rotor. However, it does tell us that flux induced by the positive sequence current components basically rotates in the same direction as the rotor and the negative sequence current components basically rotates in the opposite direction to the rotor. Since current in induced because of the change of flux, flux caused by the negative sequence current induces much more current in the rotor compared to that caused by the positive sequence current.

From the flux analysis at the center of the rotor, we have shown how each sequence component current affects the magnetic flux very differently. Zero sequence current components induce no net magnetic flux at the rotor central point. Positive sequence current components do induce magnetic flux which rotates with the rotor and does not move or change with reference to the rotor; thus, it does not induce any current in the rotor. The negative sequence components induce flux that rotates at the same speed but in the reverse direction. The changing flux in the rotor induces high heating current, which has more damaging effect to a generator or motor.

At a more general point of the rotor, the analysis of the flux is much more mathematically involved and is beyond the scope of this paper. From the case study at a point half way between the rotor center and stator conductor, we can expect that magnetic flux induced by negative sequence currents rotates in the reverse direction of the rotor. The negative sequence currents have much more damaging effect on the rotor compared to other sequence currents.

IV. Conclusion

Sequence component vectors are linear combinations of three phase phasors. They rotate in the same direction with the three phase vectors. When a generator runs in unbalanced condition, negative sequence current is most detrimental. Positive sequence current components rotate in the same direction as the rotor and induce little current in the rotor. The negative sequence components induce flux which rotates in the reverse direction of the rotor. The large changing flux in the rotor due to negative sequence currents induces high heating current, which has a more damaging effect to a generator or motor.

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